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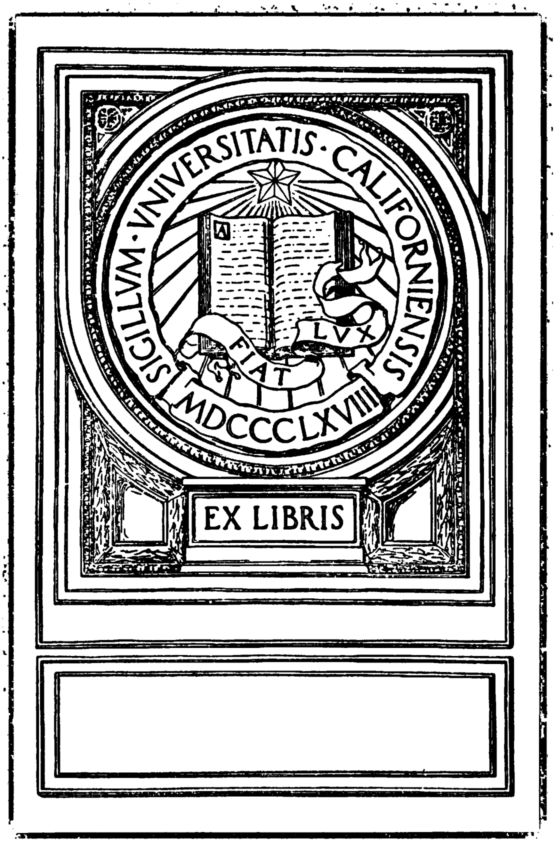
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AN ELEMENTARY TREATISE

ON

ORTHOGRAPHIC PROJECTION.

AN ELEMENTARY TREATISE
ON
ORTHOGRAPHIC PROJECTION;
BEING
A NEW METHOD OF TEACHING THE SCIENCE
OF
MECHANICAL AND ENGINEERING DRAWING,

INTENDED FOR THE INSTRUCTION OF ENGINEERS., ARCHITECTS, BUILDERS,
SMITHS, MASONS, AND BRICKLAYERS, AND FOR THE
USE OF SCHOOLS.

WITH NUMEROUS ILLUSTRATIONS ON WOOD AND STEEL.

BY

WILLIAM BINNS,

ASSOC. INST. C.E.,

MASTER OF THE MECHANICAL DRAWING CLASS AT THE DEPARTMENT OF SCIENCE
AND ART, AND AT THE GOVERNMENT SCHOOL OF MINES; FORMERLY
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AT THE COLLEGE FOR CIVIL ENGINEERS, ETC., ETC.

THIRTEENTH EDITION.

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PREFACE

TO THE THIRD EDITION.

A THIRD EDITION of this work having been called for so soon after the issue of the second would almost warrant the conclusion that my humble efforts to make clear a subject which mathematical professors prefer to shroud in mystery, have been appreciated.

It is also gratifying to think that the system I have introduced is now being successfully taught in all the Art Schools throughout the United Kingdom; and as many of the students from these schools have come under my notice, I have much pleasure in testifying to the very efficient manner in which the masters have performed their duty as teachers of Orthographic Projection and its application to mechanical and engineering drawing,—a subject which it would be desirable to introduce in an elementary form into our Parochial Schools. In support of this assertion, it is only necessary to observe that it is from such institutions that the greater portion, if not the whole, of our artisans emanate; and the description of drawing which they ought to learn is that which will enable them to understand mechanical and building construction. Hitherto such a thing has scarcely been thought of, because its importance has never been felt in the right quarter: besides, the subject appeared to be so difficult, that its comprehension by boys in a Parochial School was not in the least to be expected. This, however, is no longer the case; and I am prepared to state, that with the assistance of the Committee of Council on Education, and the co-operation of the masters of such schools, the practice of making to scale plans, sections, and elevations of mechanical structures, could, in a very short time, be brought into successful operation in every school in the kingdom.

W. B.

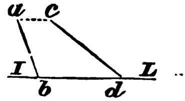
PREFACE

TO THE SECOND EDITION

HAVING, in the preface to the first edition of this work, pointed out the importance of commencing the subject of mechanical drawing at the beginning rather than at the end—the latter being the practice which is still adopted in some of our great institutions, civil and military—I feel it necessary once more to direct the attention of students and the public to the fact, that the system of teaching mechanical drawing by some of our professors is no more calculated to make a mechanical draughtsman than so much time spent in copying music would be to make a musician. What man who can use his pencil cannot copy Chinese characters? —but what amount of copying will enable him to read them? It is precisely the same with the subject in question: the various lines composing a mechanical drawing can be copied by almost any novice, but no amount of copying will enable him to understand them. It would be just as consistent to attempt the teaching of mathematics by copying algebraic formulæ, as to attempt to make a mechanical draughtsman by placing before him for imitation a number of inaccurate and distorted

French lithographs of machinery. If copying is practised at all—and it may be with much benefit in those institutions where they have not the advantage of actual machinery or models—it should be from first-class working drawings of machinery; but even this practice should not be introduced until the student has acquired a sound knowledge of "orthographic" projection, which will not only enable him to understand every line on the vertical and horizontal planes, besides something of the construction of the machine he is copying, but will also enable him to detect the errors which are so frequently to be met with in the drawings referred to. But, without a knowledge of the planes of projection, and the projection of right and curved lines upon those planes, the practice of copying is a mere waste of time, and ought, for the credit of the institution in which it is practised, to be discontinued. Another and more important reason is the welfare of the students, many of whom, since the first issue of this work, have joined the classes which I have the honour to conduct at the Department of Science and Art, South Kensington, and at the Government School of Mines, Jermyn Street; and notwithstanding they had previously spent from twelve to eighteen months in the vain hope that they were learning mechanical drawing, they could not understand what is meant by *three projections of a right line.* Of those persons who not only practise, but recommend, the system I am denouncing, I would ask whether, in the whole course of their teaching.

they ever met with a copyist who could find the actual length of a right line, of which two projections only, *a b*, *c d*, in the vertical plane were given, and also the angles which such line makes with the vertical and horizontal planes; and yet these are lines which may be found in mechanical drawings. I may even go so far as to say of those students who have, unfortunately, wasted many months in one of our first metropolitan colleges, and who have since attended my class, that when they first placed themselves under my tuition they were not able to find correctly the projections of the most elementary figures given in this work, and shown on drawing A;* whereas the amount of time they had already spent on the subject was *sufficient to enable them to make elevations, plans, and sections of machines from actual measurement.*



Before closing this preface, I would respectfully beg of those gentlemen who have objected to the principles of this work, especially those laid down for the projection of objects on the inclined plane, which I am flattered to think they have been pleased to call an invention of my own, to communicate with the author, rather than mislead their students, or to state their objections publicly, when they shall be answered.

I would also beg to express my unfeigned pleasure at the receipt of so many complimentary letters from the working class, and to say that there are none who know their requirements so well as those who have been in their position,—a circumstance which I am proud to acknowledge in my own case, as it has enabled me to lay before them the subject matter of this work, which was prepared as much for their use as for a school class-book.

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* Read also Art. 10, page 11, which gives an illustration of their knowledge of projection.

I have also to acknowledge the receipt of many applications for the second part of this work, conditionally promised in the first preface, and to say that my professional engagements, since the first issue, have prevented me from complying with their wishes, although it is my intention to proceed with it at my earliest convenience.

W. BINNS.

15, *Edith Grove, Brompton, London.*

PREFACE.

The great and increasing demand for a knowledge of Geometrical Drawing as applied to the Arts has induced me to lay before the public a course of instruction which I designed in 1846 for the Students at the College for Civil Engineers, Putney, and practised with success until the close of that institution in 1851.

During the past four years an opportunity has been afforded me by the Department of Science and Art of remodelling the subjects of this work, which is intended as a text-book for the Masters in training, and as a medium through which the Engineer, the Mechanist, the Architect, the Builder, the Mason, and the Bricklayer may acquire a knowledge of the rules and principles governing that description of drawing by which their operations are directed when constructing a machine or erecting a building.

The object of teaching Geometrical Drawing, whether applied to civil or military engineering, is to give the power of representing, by plans, elevations, and sections, the magnitude and configuration of a structure, such as a machine bridge, house, or fort; and since all such representations are governed by the same rules, a knowledge of their application is of equal importance. The design of this work is, therefore, to lay before the reader a simple and condensed course, forming the groundwork of Mechanical and Engineering Drawing; so as to place within his reach the means of acquiring a knowledge of such delineations when laid before him to work from; and also to give him the power, if only with a piece of chalk and a board, of expressing his ideas of any mechanical or engineering structure which he may have occasion to carry out in practice.

Notwithstanding that we have recently been favoured with translations from the French of Le Blanc, Armengaud, and Amouroux, there is, to the best of my knowledge, no work in the English language of a sufficiently elementary character to meet the wants of the artisan, who requires to be led step by step through a series of easy and progressive exercises on the projection of points, right lines, and simple figures; after which he may be introduced to the subject of curved lines and the delineation of more complex figures. This is the plan I have followed in the work now submitted to the public, which is intended as a basis of instruction for those whose business or profession may require them to understand geometrical delineations.

Having, in the title-page of this book, presumed to designate the system here recommended "a new method of teaching Mechanical and Engineering Drawing," it may be desirable to show wherein the novelty consists, by a comparison with the usual mode of teaching, which, so far as my knowledge extends, is from the "flat,"—that is, from copy—the practice being to lay before each student of the class a drawing of some part or parts of a structure, which he is requested to copy. This being done, another drawing, probably somewhat more elaborate, is laid before him; and the same course is pursued until he becomes tolerably expert with his instruments and brushes, and eventually is enabled to make a very creditable or even a highly finished drawing from *copy*. If, however, at the end of one or two years' practice, the copyist is asked to make an end elevation, side elevation, and longitudinal section of his black-lead pencil, or a transverse section of the box containing his instruments, the chances are that he can neither do the one nor the other. Strange as it may appear, this is a state of things which I have had frequent opportunities of witnessing in the case of students who, on entering my class, have brought excellent specimens of their own drawings from copy. The remedy in all such cases has been to commence a course of study from the very beginning; and the result has ever been satisfactory. I should therefore most earnestly recommend the student who has gone through a course of instruction from the "flat," and who is desirous of acquiring a knowledge of those principles which govern the operations of the draughtsman in the execution of his designs, not to be led astray by the impression that the projection of a single point or line, or the attempt to make a drawing of his black-lead pencil, is too insignificant for consideration because he has already made from copy a highly finished drawing, with all the shadows admirably projected, being at the same time, however, perfectly ignorant of the rules for projecting such shadows. This, it may be remarked, is the true picture of a student who had completed a course of two years' study at an institution where mechanical drawing was taught. After three months' practice with the elements of Orthographic Projection, as given in this work, the same student informed me, with great satisfaction, that his readings in Descriptive Geometry, which had previously been a task, had now become a pleasure, because he could understand them.

With the conviction that the subject of Orthographic Projection is the A B C of that description of drawing which is universally adopted as a means of representing all kinds of engineering structures, as well as articles in process of manufacture, it has been my sole aim, during the years I have been engaged in lecturing, to reduce the subject of Mechanical Drawing to a series of fixed rules and principles methodically arranged; so that the whole subject, from the projection of a point or line to the projection of the most complex object, forms, as it were, a chain of so many links,—each problem having some bearing on, or connection with, the one preceding. It will also be found that *each division* *of the subject in this work commences with the projection of a single point*, and then proceeds to the projection of a line; and that the projection of a curved line is not introduced before a full explanation has been given of the projection of right lines.

It will, therefore, be understood that such a connection of the subject renders it of the utmost importance that the first Chapter should be thoroughly mastered before the second is commenced; and so also of the Problems contained in the Chapters, which ought to be carefully drawn, as shown in the plates, drawings, and figures, but on a larger scale. The result of such a course will be an amount of practice with the instruments equivalent to that of drawing from copy, with the important advantage of acquiring at the same time a knowledge of the principles.

Another peculiarity in this new method is that of *collective teaching* by class lectures. These lectures commence with an illustration of the vertical and horizontal planes on which the objects are to be drawn. This is followed by the projection of a point or line, and afterwards with horizontal projections of simple objects, which are carefully worked out on the black board and explained to the students of the class, all of whom are engaged on the same problem, so that an opportunity is thus afforded to those who are in advance to assist those who are less fortunate. It may also be remarked that each lecture is concluded by giving out a subject for study and practice, which is to be worked out on the black board and explained by the Master at the succeeding lecture; and so on throughout the course, which extends over two sessions of five months each. Thus, with ten months' practice, the intelligent student may acquire a knowledge of those principles which will enable him to make plans, sections, and elevations of a machine or building from actual measurement. He may likewise acquire a thorough knowledge of the method as well as the power of delineating any object which he can conceive, or which can be explained to him. Moreover, his powers of conception and understanding when reading scientific works or attending lectures, which are invariably illustrated by orthographic representations, will be proportionably increased, so that the construction and action of a machine can be fully realised from such projections, without the aid of a model or reference to the machine itself.

Such, then, is a general outline of the system pursued, which is divided in the manner following:—

The First Course includes the projection of points, lines, and plane figures; plans, elevations and sections of geometrical solids upon the upper, lower, and inclined planes; sections and penetrations of cylinders, cones, spheres, and other figures; the development of lines upon plane surfaces, and their projection upon curved surfaces; and, lastly, the projection of screws. Thus far the present volume is intended to conduct the student.

The Second Course (which may hereafter form the subject of another treatise) embraces the delineation of worm, bevel, mitre, and spur wheels; the principles of the formation of the teeth of wheels, and the practical modes of constructing them by means of templets and the "Odontograph;" the cycloid, epicycloid, and hypocycloidal curves; the construction of cams, wipers, heart-wheels, and eccentrics; the projection of shadows; and the practice of making to scale drawings from actual machinery.

It is necessary to observe that as this work has been prepared with a view of supplying the artisan with an elementary course of instruction, it has been deemed sufficient to explain only the first section of the course, for if this be properly understood, it will enable those who are desirous of extending their knowledge to read and comprehend such works as "The Practical Draughtsman's Book of Industrial Design," edited by William Johnston, Assoc. Inst. C.E., and "The Engineer and Machinist's Drawing Book," published by Blackie and Son.

I may here mention that I am indebted to Le Blanc's "Dessin des Machines" (dated 1836), and to Nicholson's "Orthographic Projection" (dated 1837), for some of the figures which are given in the latter part of this work, and employed as practical illustrations of the problems laid down in the earlier part.

In teaching the subject of Orthographic Projection it is necessary to make frequent reference to such of the preceding problems as bear upon or are connected with the subject under consideration; it is therefore possible, in my anxiety to preserve that connection which is one of the characteristics of this work, that some repetition may occur, notwithstanding every care has been taken to avoid it by dividing the subject into Articles, which the student is requested (by references) to read again, and work with his pencil, until the difficulties he meets with are overcome.

WILLIAM BINNS.

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ORTHOGRAPHIC PROJECTION.

CHAPTER I.

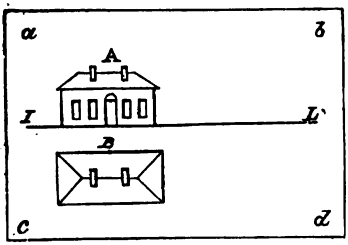
ILLUSTRATION OF THE PLANES OF PROJECTION.

IN commencing the study of orthographic projection, and its application to model, mechanical, and engineering drawing, it will be necessary for the student to provide himself with the following articles:—A drawing board of convenient size, say 24 inches by 17 inches, which will take half a sheet of imperial paper; a T square for the same; one 8-inch set square, angle 45°; one 9-inch set square, angles 30° and 60°; half-a-dozen drawing pins; an HH black-lead pencil; and some imperial cartridge or drawing paper: all of which articles may be purchased of any artists' colorman. With regard to instruments, only a pair of compasses with good sharp points will be required until the student has gone through the projection of rectangular objects and arrived at that part of our subject which introduces him to the projection of curved lines, when he will be in a position to decide if the progress he has made warrants the purchasing of a set of drawing instruments.

ART. 1.—The *planes* upon which the objects are to be represented are called the planes of projection, and constitute the first subject for consideration They are three in number, namely, the *vertical plane*, the *horizontal plane*, and the *inclined plane*. Those planes, however, which demand our immediate attention, are the vertical and horizontal planes.

As a familiar illustration of these planes, let the student imagine himself looking through a window at a house opposite. Such a view of the house would be in the vertical plane, as represented at A, Fig. 1. Now let him suppose that he is looking down upon the house from some elevated position. This view would be in the horizontal plane, as shown at B, Fig. 1, which is a plan of the house. It may be stated, as a general rule, that *all elevations of objects are to be represented in the vertical plane, and all plans of objects in the horizontal plane.*

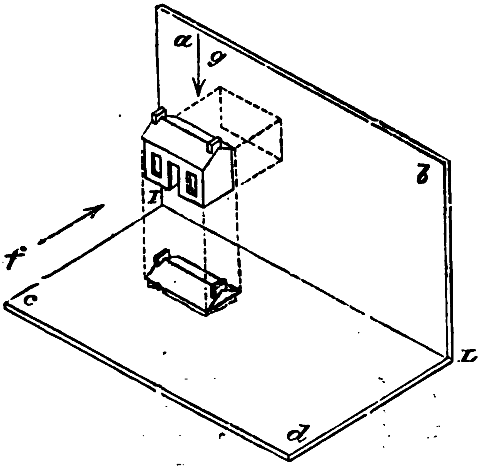
Fig. 1



ART. 2.—The vertical and horizontal planes are generally divided by a line, called the ground line, or *intersecting line* of the two planes of projection. Let *a c d b* represent a sheet of drawing paper upon which any object is to be delineated. In any convenient part of the sheet draw I L, the intersecting line: then will *a* I L *b* be the vertical plane of projection, and *c* I L *d* the horizontal plane. Therefore, since the planes are to be considered as vertical and horizontal, they must necessarily be at right angles to each other. An illustration of this is exhibited at Fig. 1*a*, which is a drawing in isometrical perspective of the two planes bent at right angles to each other at I L, the intersecting line of Fig. 1. It will sometimes happen when attentively examining Fig. 1*a* that the bend or angle I L will successively assume two appearances, that of an internal and external representation. This effect is most likely to be observed if the eye

be directed along the intersecting line from I to L, when the angle *a* I *c*, or *b* L *d*, will appear to change from internal to external in rapid succession. This circumstance, although it may be somewhat amusing, materially affects our proposition, which supposes *a* I L *b* to be the vertical plane, with the arrow *f* indicating the direction of the visual rays when viewing the elevation of the house, and *c* I L *d* to be the horizontal plane, with the arrow *g* pointing out the direction of the visual rays when viewing the plan of the house. In isometrical drawing, however, we have a plan and elevation in one view, and although its application may not be strictly correct in this particular case, it may be useful in demonstrating the proposition of the two planes of projection.

Fig. 1*a.*

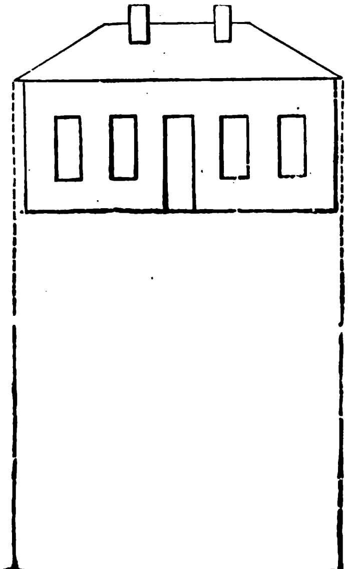


Art. 3. — Should the reader still experience some difficulty in realising the relative position of the planes of projection, it is hoped that much of that difficulty will be removed by perusing the two following pages,—the book being turned for that purpose at right angles to its present position.

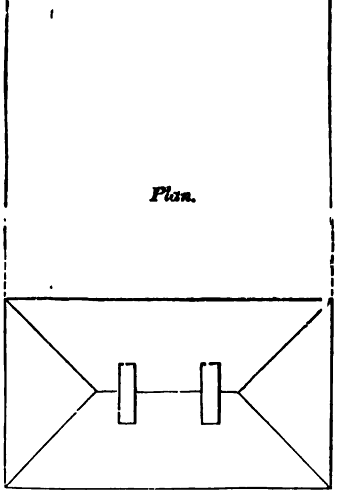
Supposing this book to be laid open on the table, the two pages exposed may be said to represent the sheet of paper on which a drawing is to be made, and the line formed by the folding at the bottom of this page will represent the intersecting line of the two planes of projection. Therefore that portion of the surface which is above the intersecting line is called the upper or vertical plane; and it is upon this plane or part of the sheet that all elevations are to be delineated: such, for example, as the elevation of a house, a machine, or any other object, as in the accompanying illustration, Fig. 2.

Fig. 2

Elevation.



Again, that portion of the surface which is below the intersecting line is called the lower or horizontal plane of projection; and it is upon this plane that all plans of objects are to be drawn.

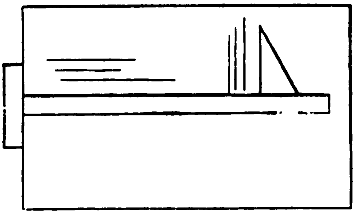


If the student will now raise the above page with the left hand, as in the act of closing the book, so that the two pages are at right angles to each other, he will have a correct representation of the relative positions of the two planes of projection.

Whilst the upper page is retained in the position just mentioned, we will suppose a model, of the exact dimensions of the drawing, to be held in the right hand immediately above the plan, and exactly opposite the elevation. If, under these circumstances, imaginary lines be drawn from the corners of the roof, chimney, and other parts, at right angles to the two planes of projection, such lines will represent the visual rays, which in orthographic projection are *parallel to each other;* and if the points where such visual rays touch the two planes of projection be united by right lines, such lines will represent the orthographic projection of a house in plan and elevation.

ART. 4.—We will now consider the student to have prepared his drawing board, and fastened down the sheet of paper with drawing pins, which will answer every purpose until he begins the projection of elaborate figures, when it may be desirable to secure the paper on the board with glue, by damping its surface with water by means of a sponge, and gluing the edges to the drawing board. Previous to taking the subject of the next chapter into consideration, attention must be paid to the following instructions in the use of the T square and set square:—Place the stock of the T square against the *left-hand* edge of the drawing board, and hold it firmly in that position with the left hand. Then, by sliding the T square along the edge of the board, and applying the pencil to its upper edge, parallel lines may be drawn across the paper from left to right. These lines we will denominate horizontal lines. And by applying the set square to the upper edge of the T square, as shown in Fig. 3, and sliding it along such upper edge, parallel lines may be drawn at *right angles* to the horizontal lines, that is, presuming the set square to be correct, in which case they will be at right angles to the parallel horizontal lines, and may therefore be regarded for the present as vertical lines.

Fig. 3.



CHAPTER II.

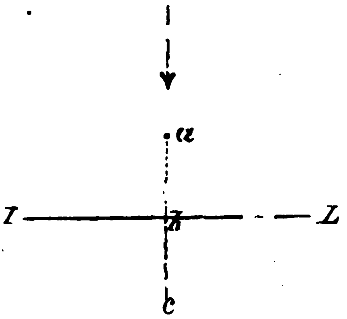
PROJECTION FROM THE UPPER TO THE LOWER PLANE.

PROBLEM I.

Given the position of a point or line in the vertical plane, to find the projection thereof in the horizontal plane.

ART. 5.—Let I L, Fig. 4, be the intersecting line of the two planes of projection, and *a* the given point, which is supposed to represent a line projecting from the vertical plane a distance of 3 inches. Then, with the T square and set square applied as directed for drawing vertical lines, let fall a perpendicular line *a b c*, from the point *a*, in the vertical or original plane,* making *b c* equal to the supposed length of the line *a*. Then will *b c* be the projection of the line *a*, as viewed in the direction indicated by the arrow,

Fig. 4.



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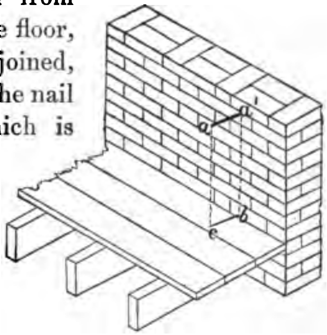
* The term *original plane* is used to designate that plane, whether vertical, horizontal, or inclined, which contains the points, lines, or figures to be projected.

which, in this and the following propositions, we will suppose to represent the direction of the visual rays, or the direction in which the figures to be projected are viewed.

ART. 6.—The difficulty experienced by many students in thoroughly comprehending the foregoing problem has frequently obliged the writer to dwell upon it at some length; and when it is considered that there is scarcely a problem from this page to the end of the book in which the first does not find a place, it will be admitted that a knowledge of the principles which govern the projection of a single point or line is of the utmost importance. In order that the first problem may be clearly understood before the second is commenced, we will imagine the student to be looking at a wall, in which there is driven a nail or piece of wire, immediately in front of or in a direct line with the eye. It is manifest that such a view would be correctly represented by a point, dot, or mark, as shown at *a*, Fig. 4. But if the observer stood upon the wall, he would on looking down see the length of the nail, or, rather, the amount of its projection from the face of the wall. If an imaginary line be now drawn from each end of the nail to the floor, as *a′ b*, *a c*, and *b c* be joined, then will *b c* be a plan of the nail as shown at Fig. 4*a*, which is an isometrical view of the subject of the first problem, Fig. 4. We must now take our leave of this description of drawing, and direct attention to the following article.

Fig. 4*a*.



ART. 7.—*The plan of any given point or line will in all cases be obtained by letting fall therefrom a vertical line, and making the length of the projected line in the horizontal*

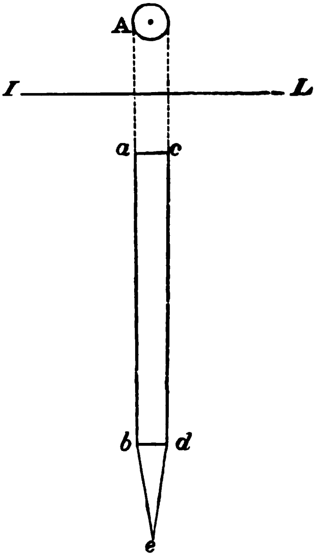
plane equal to the supposed length of the line represented by the given point. If the point represent a line of 3 inches, the plan of the line will be 3 inches; and so on for any other length.

PROBLEM II.

To find the end elevation and plan of a black-lead pencil.

ART. 8.—If such an object were held in a horizontal position, with the point directed to the eye, its appearance would be that of a circle with a dot in the centre, as shown at A, Fig. 5, which may be called an end elevation. It is now required to find the plan, or the appearance which a pencil would present, if viewed as directed for the projection of a point. From the two sides (or the ends of an imaginary horizontal line representing the diameter) let fall vertical lines, *a b*, *c d*, making them equal to the length of the pencil. Draw *a c*, forming the end of the pencil, and a line, *b d*, forming the base of the point, which is completed by drawing two lines, *b e*, *d e*, meeting each other at a suitable distance from the base. Then will *a e c* represent a plan of the pencil.

Fig. 5.



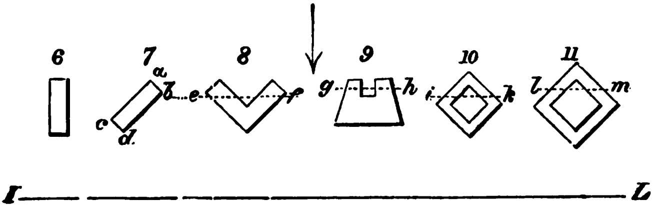
It is not absolutely necessary that the lower view, *a e c*, should be in the horizontal plane, as the same might as consistently be drawn in the vertical plane; but of this more will be said hereafter; for the present it will be convenient to consider the upper figure as an end elevation, and the lower figure as a plan.

PROBLEM III.

The end elevation of an object being given, to find the plan thereof.

ART. 9.—The following Figures, 6, 7, 8, 9, 10, 11, represent the end elevations of a series of objects of which it is required to find the plan or appearance they would assume if viewed in the direction indicated by the arrow, which applies to all.

Figures

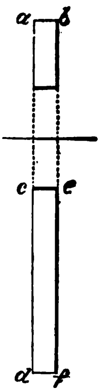


Presuming the student has acquired a knowledge of the projection of a point or line, he will now have to determine the number of lines composing each figure, and which of those lines would be seen if the figures were viewed in the direction indicated.

In order to avoid repetition, the student is requested to bear in mind that when horizontal lines are mentioned, such lines are to be drawn with the **T** square, and vertical lines with the set square.

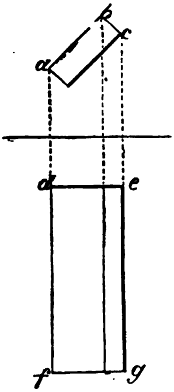
We will now suppose that each object in the examples Figs. 6 to 11 is three inches long, and that Figs. 6 and 7 represent the end elevations of two beams of timber, one of which is vertical as regards its sides, and the other is inclined to the vertical. It is evident, with respect to Fig. 6, that the two upper corners would form lines, and their projection would be found as described in Art. 5. A plan of this figure will therefore be obtained by drawing from the points *a* and *b*, lines *c d*, *e f*, Fig. 6*a*. Then will *c d* be the projection of the point or line *a*, and *e f* the projection of point or line *b*. Again, line *d f*, in the plan, will be the projection of the line *a b* in the elevation, and will represent that end of the object which is nearest the eye of the observer. In like manner, the line *c e* will be the representative of the opposite end of the object, or that most remote from the eye of the observer.

Fig. 6a.



With regard to Fig. 7, it is also evident that the line formed by the lower corner or angle will not be seen if the object is viewed from above. The plan will therefore be obtained by drawing vertical lines from the points *a*, *b*, *c*, Fig. 7*a*, and uniting such lines by drawing the horizontal lines *d e*, *f g*, at such distance asunder as will exactly equal the length of the object. Then will the figure or projection *d f g e* be a plan of the beam of timber, of which *a b c* is an end view.

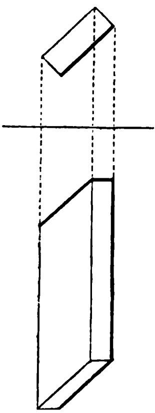
Fig. 7a.



Art. 10.—It may be desirable in this place to guard the reader against an error committed by those who have had some practice in perspective drawing, when attempting an orthographic projection of Fig. 7, which they invariably draw as shown in Fig. 7*b*. This is incorrect, inasmuch as it is neither a scenographic representation nor an orthographic representation of the object. To understand the reason of this it will be necessary to consider the difference betwixt perspective drawing or sceno-

graphic projection and orthographic projection. In drawing objects according to the principles laid down for perspective, the eye is imagined to be stationed in one particular place, called the point of sight, from which all the visible parts of the figure are supposed to be seen. The lines representing such parts are then made to vanish in some other point in the plane of the picture, called the vanishing point. But with orthographic projection the case is very different, inasmuch as the eye is supposed to be in a direct line with every part viewed, or, in other words, to move over the object in such manner as to be directly opposite to every part represented. The visual rays are therefore parallel; whereas in perspective they converge to a point, which is called the point of sight.

Fig. 7b.



Art. 11.—Having thus cautioned the reader against any attempt at perspective in obtaining the plan of Figs. 8, 9, 10, and 11, it is only necessary to add that Fig. 8 is an end view of two pieces of timber joined together in the form of a trough or **V**; Fig. 9, an end view of a wedge-shaped piece of wood with a groove along its upper surface; Fig. 10, a square tube; and Fig. 11, a solid rectangular block or prism of the following dimensions,—viz., the larger part being 1½ inch long and 1 inch square, and the smaller part 1½ inch long and ¾ of an inch square, making together a figure equal in length to the preceding. It is now required to find the plan of these figures.*

Art. 12.—In mechanical as well as in architectural and other descriptions of drawing, it is frequently necessary that some part or parts of the object should be "drawn in section." This is the technical phrase, and implies the division of such part or parts from the remainder of the object. The representation of the parts so divided is said to be in section, and may be distinguished from other parts in a mechanical drawing by a series of equidistant parallel lines, termed section lines, generally drawn with the set square at an angle of 45°, as shown in Fig. 12, which is a section of Fig. 7, taken through the line *a b*: this line is commonly called the *line of section*.

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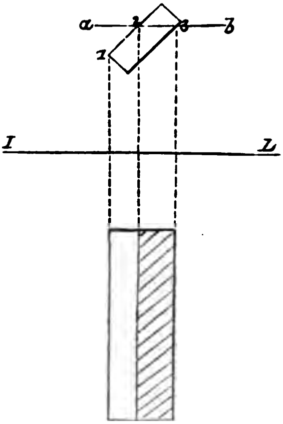
* Before referring to Drawing A, in which the projection of the foregoing figures will be found, the student is earnestly recommended to make the best attempt in his power with the above examples, after which he may consult the Drawing, to test the accuracy of his work.

Fig. 12.



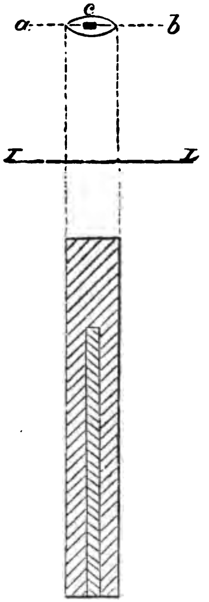
The reader is requested to bear in mind, with regard to this and the following figures, that the upper portion, or that part of the figure which is above the line of section, is supposed to be removed, and that he is looking upon the remaining part in the direction indicated by the arrow, ART. 9. It will, therefore, be evident that to obtain the section, Fig. 12, the lines 1, 2, 3, must be projected as indicated by the dotted lines. Having drawn these lines, and also those representing the ends of the figure (as directed by ART. 9), it is only necessary to determine what part of the figure is in section,—such part being that only through which the line of section *a b* passes; namely, from 2 to 3. Therefore, the corresponding part in the plan must be drawn in section in the manner described and as shown in the plan, Fig. 12, which is called a *sectional plan.*

The student may now proceed with the sectional plans of the remaining figures, the lines of section being drawn through the points *e f*, *g h*, *i k*, and *l m*, see Figs. 8 to 11.

and when complete, the work may be compared with the sectional plans in Drawing A.

Art. 13.—The object of a section is to show the internal configuration or arrangement and combination of parts of which anything is composed. As a familiar illustration of the application, let us suppose that a manufacturer of black-lead pencils has to make a number of those articles to order. Now, since there are pencils of various forms and lengths, some with lead throughout, and others with lead extending scarcely more than half the length of the wooden part, it will be necessary in giving the order to explain these things, as well as any peculiarity in the shape of the wooden part, which, for example, we will suppose to be of an elliptical form. The most convenient mode of describing this little object would be by the aid of a drawing, showing an end view and longitudinal section (that is, a section throughout its length), as in Fig. 13, where *c* represents the end view of one of that class of pencils whose locomotive propensities over the drawing board and on to the floor are interfered with by its flat or elliptical form. In this figure, the breadth, thickness, and shape of the wood, as well as the strength or thickness of the lead, are clearly exhibited. The respective lengths of the wood and lead are given in the section, which is taken through the line *a b*,—the upper part of the figure being removed.

Fig. 13.



Art. 14.—The reader will perceive that the section

lines in Fig. 13 are drawn in opposite directions. This plan is adopted in the sectional representations of objects consisting of several parts, for the purpose of more readily distinguishing one part from another. It not only improves the appearance of a drawing, by relieving it of that sameness which would result from drawing all the section lines in one direction, but it is absolutely necessary when delineating objects composed of two or more parts, as will be explained hereafter.

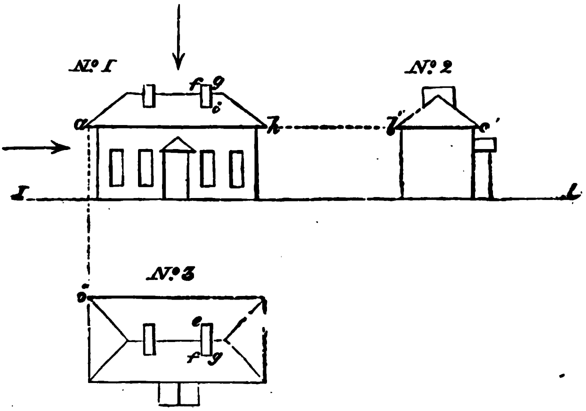
Art. 15.—It will be observed, with regard to the foregoing figures, that we have made use of two planes of projection. These planes are sometimes called the *upper* and *lower*, as well as the vertical and horizontal. In Drawing A, the figures contained in the first rows, which are said to be end elevations of certain objects, are in the upper plane; and the figures of the second row, which are said to be plans of those objects, are in the lower plane. Now the sectional figures of the third row, being also in the lower plane, are called *sectional plans* of the objects in the first row. It will likewise be remembered that in order to obtain the projection of any given point or line from the end elevation, a vertical line, as *a b c*, Fig. 4, was drawn from such point, and the projected line was called a plan of that point, because of its being below the original and in the lower plane. This is called *projection from the upper to the lower plane.*

CHAPTER III.

PROJECTION IN THE UPPER PLANE.

Having explained the projection of points or lines in the lower plane, called plans, we must now direct attention to the projection of figures in the upper plane, termed elevations.*

Fig. 14.



ART. 16.—Let No. 1, Fig. 14, represent the side elevation of a cottage; No. 2 an end elevation, and No. 3 a plan thereof. Again, let the point *a*, No. 1, represent the end of the line forming that edge of the roof which extends along the end of the house. If from the point *a* we let

fall a line, *a b c*, and make *b c* equal in length to the width of the roof, then the line *b c* will be a *plan* of the point or line *a*. Again, if from the point *a*, a line, *b′ c′*, No. 2, be drawn parallel to the ground or intersecting line, I L, and *b′ c′* be made equal to the supposed length of the line represented by the point *a* (which it has been said is equal to the width of the roof), then *b′ c′* will be an *elevation* of the point or line *a*. In like manner we obtain *e f*, No. 3, being the plan of the point or line *f*, No. 1; and *g i*, No. 1, the elevation of the point or line *g*, No. 3.

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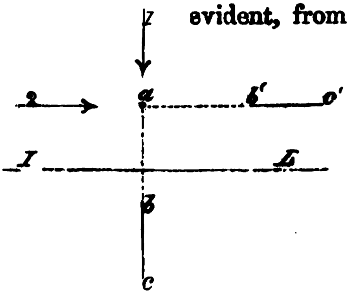
* In the illustration of plans and elevations of right lines in the two planes of projection, the teacher will find a model or large diagram of some familiar object, such as Fig. 14, of great service.

ART. 17.—It has been stated that if a given line in the vertical plane, as *a*, Fig. 15, be viewed in the direction of the arrow 1, its projection, *b c*, will be a *plan* of *a* (ART. 5). It will also be evident, from the preceding article, that the original point or line may be viewed in the direction of the arrow 2; in which case the projection would be obtained as follows: — From *a* draw *b′ c′*, parallel to the intersecting line; make *b′ c′* equal to the supposed length of the line represented by point *a*: then will *b′ c′* be the projection of the point *a*. This is called *projection in the upper plane;* and all objects in that plane are called *elevations:* therefore *b′ c′* is an *elevation*, and *b c* a *plan* of the original point or line *a*. If the reader will bear the following article in mind, he will save himself much trouble in turning back and referring to that which ought to be engraven on his memory.

Fig. 15.



ART. 18.—*The elevation of any given point or line will in all cases* (unless it is otherwise stated), *be in a right line drawn from such point or line parallel to the intersecting line;* the projected line being made equal in length to

the supposed or actual length of line represented by the original point or line.

ART. 19.—In finding the projection of any given line represented by a point, the first question which suggests itself is, what is the length of line represented by the original point? or, in other words, *what is the distance from the original point to the point beyond it?* Simple and commonplace as this expression may appear in a work intended to illustrate the practical application of the principles of descriptive geometry, the question just proposed is one which must be answered by the student before he can lay down the projection of any one given line. We shall not, therefore, offer any apology for its frequent use in this work, but proceed with an explanation.

ART. 20.—The point *a*, Fig. 15, is supposed to represent a line; and since a line must have length, there will be a certain distance from *a* to the point beyond *a*, or the other end of the line. Now that end of the line which is beyond *a* is evidently the farthest from the eye; therefore the letter or figure used to denote a point or line, will generally refer to that end of the line nearest to the eye. It must also be borne in mind, with regard to the plan or elevation of a line, *that the end of the projected line which is farthest from the original point will always represent that end of the original line nearest the eye.* Therefore *c′* is the elevation of the point *a*, and *b′* the elevation of the point beyond *a*. In like manner *c* is a plan of the point *a*, and *b* a plan of the point beyond it, or that end of the line most remote from the eye of the observer.

ART. 21.—We would here impress upon the teacher that it is scarcely possible to give too many illustrations of the projection of a single point or line; and the more familiar those illustrations can be made the better. When it is remembered that the whole subject of orthographic projection, or the practice of making mechanical drawings, is founded on a knowledge of the projection of a single point or line, and that a finished outline drawing of a machine is nothing more nor less than a combination or multiplication of the principles contained in the foregoing paragraphs, it will be manifest that the projection of a complex object can never be satisfactorily accomplishe until a thorough knowledge of the projection of a poin on the lower and upper planes has been acquired.

PROBLEM IV.

Given the elevation of a cube, required the projection thereof.

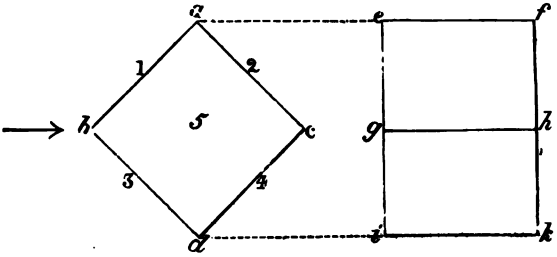
ART. 22.—Let *a b d c*, No. 1, Fig. 16, represent the elevation of a cube, of which it is required to find the projection, or the appearance it would present if viewed in the direction indicated by the arrow.

Before we proceed with the projection of this figure, it will be desirable to state that a cube is a solid composed of six equal sides or faces (viz., 1, 2, 3, 4, 5, and the face beyond 5), with twelve boundary lines. The student must not look upon No. 1, Fig. 16, merely as a square or superficial figure, but as a solid; and, in so doing, he will find that the face 5 is bounded by four lines, *a b*, *a c*, *b d*, and *c d*. The face beyond 5 is also bounded by four lines, corresponding to *a b*, *a c*, *&c.* Again, the two faces 5 and 6 are connected by four lines, *a*, *b*, *c*, *d*, represented by the corners of the square, thus making twelve boundary lines, equal in length to each other—that is, from *a* to the point beyond is equal to *a b* or *a c*. It is now required to find the elevation or projection of the cube as seen in the direction of the arrow.

Art. 23.—From *a* draw *e f*, making *e f* equal in length to *a b* or *a c:* then will *e f* be the elevation of the line represented by point *a*. From each end of the line *e f* let fall a vertical line *e i*, *f k;* from *b* and *d* draw lines *g h*, *i k:* then will *e i k f* be the required projection of the cube.

Fig. 16.

No. 1. No. 2



Art. 24.—The student may now exercise his powers of conception by looking at No. 2 as a solid, and endeavouring, while so doing, to realise the twelve boundary lines, and to comprehend that the space bounded by the lines *e g h f*, is an elevation of that face of the cube numbered 1, and the space bounded by *g i k h*, an elevation of the face numbered 3. Also that the vertical line *f k*, No. 2, represents that face of the cube No. 1 nearest to the eye of the observer, or the surface between *a* and *d;* and in like manner, that the vertical line *e i* is that face of the cube which is farthest from the eye (Art. 20). It must likewise be understood that *h* is the projection of the point *b*, and that there is a certain distance from *h* to the point beyond it, equal to *b c;* furthermore that *f h* is the projection of line *a b*, and that *e g* is the elevation of a line beyond *a b*. *f h* may also be said to represent the lines *a b* and *a c*, because the line *f h k* represents the four lines *a b*, *a c*, *c d*, and *b d*.

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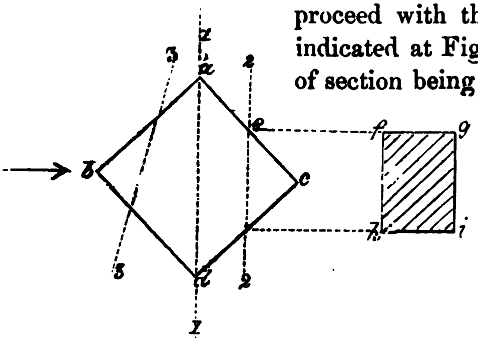
ART. 25. — As doubtless the student now understands the relationship of the lines forming the two elevations of a cube, we shall proceed with three sections, as indicated at Fig. 17,—the lines of section being drawn through the points 1 1, 2 2, and 3 3; presuming in all cases where a *sectional elevation* is to be produced, that the part of the cube which is on the left hand of the line of section is removed.

Fig. 17.



Before the reader proceeds any further with the text, he is recommended to complete his drawing of the three sections, and then consult Drawing A, and the following paragraphs respecting them.

ART. 26.—Section through the line 1 1. This section is nothing more than No. 2, Fig. 16, with the line *g h* omitted, and section lines drawn as explained in ART. 12.

ART. 27.—Section through the line 2 2. It is somewhat remarkable, after an exposition like the preceding, that many students, when getting out this section, measure the distance from *c* to *e*, and lay that down as the length of line represented by the point *e*, as shown at *f h i g*, Fig. 17. In cases where this occurs the following questions may be asked :—

1st.—What is the distance from *a* to the point beyond it ?

Ans.—From *a* to *b*.

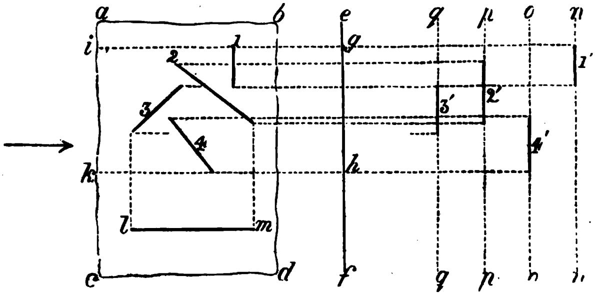
2nd.—What is the distance from *c* to the point beyond it?

Ans.—The same as from *a* to *b*.

Then *a c* represents an edge-view of one of the upper faces of the cube; and a line drawn across that face at right angles to the plane of projection must be equal to one of the boundary lines. Therefore the section *f h i g* is incorrect. [See Drawing A.]

Art. 28.—As many of our students have a predilection for representing the cube and other objects scenographically, for the reason that this method of delineating objects requires less mental labour than a system of drawing which supposes one line to be the projection of any number of lines, it may be of service to give some further explanation of this orthographic paradox. Let 1 2 3 4, Fig. 18, represent the elevation of four lines, drawn at any given angle with the intersecting line, but *parallel* to the plane of projection. If we now suppose each line to be the same distance from the point, or rather points, of sight (for in orthographic projection there are as many points of sight as there are points to be represented, because the visual rays are parallel—Art. 10), they must necessarily be in the same plane. Draw upon a

Fig. 18.



slip of paper, *a c d b*, any number of lines, as 1, 2, 3, 4, and hold the paper in a vertical position before the eye. If the paper be now turned one-fourth of a revolution, on *b d* as an axis, we shall have an edge-view or elevation of the plane on which the lines are drawn; and its representation will be a vertical line, *e f*: therefore *e f* will be the projection of *a c d b*, or a plane at right angles to the plane of projection. With this ocular demonstration before us, we can readily imagine the sheet of paper or right line *e f* to contain the four lines which have been drawn upon it. Moreover, if lines be drawn parallel to the intersecting line, through the highest and lowest points in the figure, as *g* 1, *h* 4, and produced to *i* and *k*, then will *g h* be the projection or edge-view of that portion of the sheet from *i* to *k* on which the four lines are drawn: therefore *g h* is not only the true projection of the lines 1, 2, 3, 4, but of the sheet of paper also. It is likewise manifest that a plan of the four lines would in like manner be represented by the right line *l m*, supposing *k h* to represent the intersecting line of the two planes of projection.

Art. 29.—From the above illustration we may deduce the following important proposition:—

That whatever angle a right line may make with the intersecting line, providing it be parallel to the original plane, the projection of such line in that plane will be in a right line drawn perpendicularly to the intersecting line, and the plan of such line will be parallel to the intersecting line.

Art. 30.—We will now suppose the four lines in Fig. 18 to be in the vertical plane, but not at the same distance from the eye; in other words, each line shall be in a different plane which is parallel to the original plane. Let the space between these imaginary planes be half-an-inch; and let line 1 be in the first plane, or that nearest the eye; line 4 in the second plane; line 2 in the third, and line 3 in the fourth plane. Required the elevation or projection of the four lines when viewed in the direction indicated by the arrow.

Draw *n n*, *o o*, *p p*, *q q*, parallel to *e f*, and half-an-inch apart. *n n* will therefore be the projection of the first plane, because it is nearest to the eye [read Art. 20]; and the projection of line 1 in that plane will be found as described in Art. 18. In like manner the projection of line 4 will be 4^1; of line 2, 2^1; and of line 3, 3^1. Line 1 being a vertical line, its projected length will be equal to the original line. Lines 2, 3, and 4 being inclined or foreshortened when viewed in the direction indicated by the arrow, we simply get their apparent lengths.

Art. 31.—From this example we make the following deduction:—*That an original plane, whether vertical, horizontal, or inclined, may be supposed to contain any number of planes, and each plane any number of lines.*

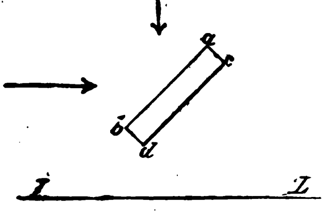
Art. 32.—It was stated at the commencement of this work that all elevations should be drawn in the vertical plane, and all plans in the horizontal plane. The writer is desirous of impressing this upon the student, from the fact that he has sometimes met with highly-finished mechanical drawings in which the plan of the machine has been above the elevation; whereas the rule is to place the elevation above the intersecting line and the plan below; although it will be shown in a subsequent chapter that it is sometimes convenient to give an elevation in the lower plane; but all such cases should be looked upon as exceptions to the rule.

Problem V.

Given the elevation of a rectangular block, to find the projection thereof in the upper and lower planes.

Art. 33.—Suppose Fig. 19 to represent a block of wood $\frac{1}{4}$ of an inch thick and 1 inch square,—that is to say, the line represented by the point *a*, or distance from *a* to the point beyond, is equal to *a b*, or 1 inch. Required an elevation and plan of the figure, which makes an angle of 45° with the intersecting line.

Fig. 19.



It will be observed that the above figure is nothing more than a repetition of Fig. 7*a*, with the addition of an elevation, which the student will readily succeed in obtaining if he thoroughly understands the projection of a point or line in the upper and lower planes. (Art. 17.)

Art. 34.—Presuming the plan and elevation to have been worked out, it should be remarked that the plan bears the same relation to the plane upon which it is projected as the elevation does to the intersecting line;—*i. e.*, the upper and lower faces of the figure in the plan, as well as its upper and lower edges, make an angle of 45° with the plane upon which it is projected, namely, the horizontal plane; and the same faces in the projected elevation of Fig. 19 will form an angle of 45° with the vertical plane of projection. It may be further stated that the front edge of the block (bounded by the lines *a b d c*) and the corresponding edge beyond it are parallel to the vertical plane; but the remaining edges (represented by the lines *a c*, *b d*), and the two faces *a b* and *c d*, are at right angles to that plane.

Art. 35.—From the above article it follows *that any plane represented by a single right line must necessarily be at right angles to the plane on which it is projected.*

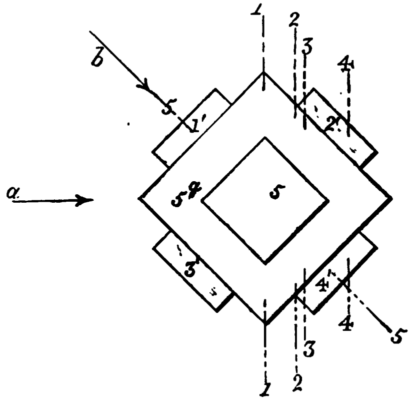
The foregoing propositions may be illustrated by the teacher with a piece of white cardboard, 5 or 6 inches square, applied with one of its edges against the blackboard.

We shall now consider the elevations and sectional elevations of a cube (Art. 21) in combination with the rectangular block just explained (Art. 33), supposing that the student has worked out the projection of both subjects.

PROBLEM VI.

Given a cube with a block on each face, to find the elevation and sectional projections thereof in the upper plane.

Fig. 20.



ART. 36. — Let Fig. 20 represent a cube, upon each face of which, and in the centre thereof, there is a rectangular block. Required the elevation or representation of the cube and blocks when viewed in the direction indicated by the arrow *a*.

Before we proceed with the projection of this figure, it may be necessary to remind the student that a cube has six faces, and consequently there must be six blocks: five only (1^1, 2^1, 3^1, 4^1, 5) are seen in Fig. 20, the sixth block being on the other side of the cube and directly opposite to block 5. It must likewise be understood that all the blocks are of the same thickness;—that is to say, the blocks 5 and 6 stand out or project from the face of the cube to the same extent as the blocks 1^1, 2^1, 3^1, and 4^1. In other words, the face of block 5 is parallel to the plane of projection; the face of the cube 5^a is also parallel to the plane of projection; and the distance between these two faces is equal to the thickness of the block 1^1 or 2^1. It must also be borne in mind that the blocks 1^1, 2^1, 3^1, and 4^1 are in the centre of the respective faces of the cube on which they are shown;—that is to say, if the cube were viewed in the direction indicated by the arrow *b*, its appearance would be precisely the same as that which is presented to the eye of the reader, who is now requested to produce the elevation when viewed in the direction of the arrow *a*. If he have succeeded in comprehending the foregoing examples, it is hoped that he will not fail in his attempt to obtain a correct elevation of Fig. 20; after which, he may proceed with the five sectional elevations on the lines 1 1, 2 2, 3 3, 4 4, 5 5.

ART. 37.—It will be observed that, to avoid confusion, the lines of section have not been drawn through the figure. Let it be understood, however, that each line of section is to be drawn in full before the section is commenced, and that the part of the figure which is on the left hand of the line of section is supposed to be removed, the remaining portion being viewed as indicated: for example, in the section taken through 5 5, the lower half is supposed to be removed, and a sectional elevation of the upper half is required.

Presuming that the student has made an attempt at the work laid before him, we shall now proceed with an explanation of the elevation and the third section,—first remarking that the accuracy of his work may be tested by a comparison with Drawing B, on which all the sections will be found.

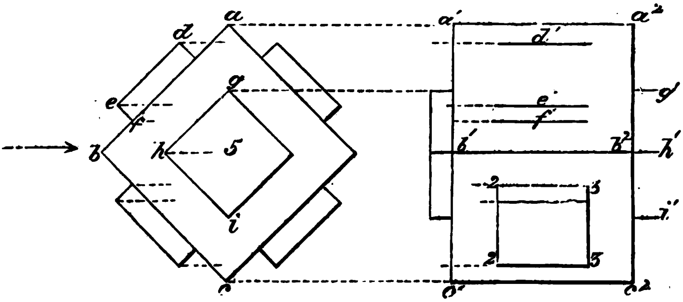
Mode of getting the elevation of the cube when viewed in the direction indicated by the arrow.

ART. 38.—From the points *a*, *b*, *c*, Fig. 21, draw lines $a^1\,a^2$, $b^1\,b^2$, and $c^1\,c^2$, making $a^1\,a^2$, &c., equal in length to one of the boundary lines of the cube; and from the points $a^1\,a^2$ let fall vertical lines $a^1\,c^1$, $a^2\,c^2$. Then will $a^1\,c^1\,c^2\,a^2$ be an elevation of the cube (ART. 23).

Now, it has been shown that the distance from *d* to the point beyond it, or length of line represented by the point *d*, is equal to *d e*, because the block is square; and it has been stated that the several blocks are in the centre of the respective faces of the cube: therefore the elevation of the points or lines *d* and *e* will be found by drawing lines

d^1, e^1, from those points, and placing them in the centre of the face of the cube,—that is to say, equidistant from the lines a^1 c^1, a^2 c^2, representing the *lateral faces* of the cube.

Fig. 21.



The student must also understand that when two planes intersect each other, the point of intersection will form a line (Art. 11, Fig. 8); consequently there will be a line at the point *f*, because we have the lower edge of the block *e f* meeting the face of the cube: therefore from *f* draw the line f^1. If vertical lines be now drawn from each end of the lines d^1 and f^1, as shown by the lines 2 2, 3 3, on the lower face of the cube, the projection of the block will be complete. A very common error is that of drawing the lines d^1, e^1, f^1 across the face of the cube. We have now to find the elevation of block 5 and the one beyond it on the opposite face of the cube.

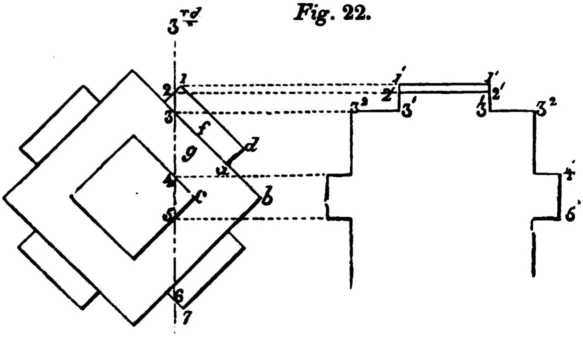
Art. 39.—As it is desirable in a work of this description to avoid as much as possible the use of figures and letters of reference, the student should remember that the lines a^1 c^1, a^2 c^2, will in future be said to represent the *lateral faces* of the cube. In like manner, the lines 2 2, 3 3, represent the lateral edges of the block. It will be observed that the blocks which require to be represented are those standing out from the lateral faces of the cube. The face of the block numbered 5 is parallel to the plane of projection, and the face of the cube, of which it forms a part, is also parallel to the plane of projection. Now the projection of all planes which are parallel to the original plane must be right lines (ART. 28); and the distance between the face of block and face of cube will be equal to the thickness of block. Therefore, from the points *g*, *h*, *i*, draw lines g^1, h^1, i^1, in length equal to the thickness of the block; through g^1, h^1, i^1, draw a vertical line to form the lateral face of the block (as shown in the representation of the sixth block on the opposite side of the cube); and the projection of the cube, with blocks, will then be obtained.

ART. 40.—In considering the sections of the cube, the student will be thrown to a considerable extent upon his own resources, inasmuch as an explanation of each section would be attended with an unnecessary amount of repetition. We accordingly purpose to give a full description only of the third section of this figure, and of the fifth section of the succeeding figure, representing a hollow cube.

Commencing at point 1 of the third section, Fig. 22, we get the line $1^1\ 1^1$, which is equal in length to one side of the block; and from point 2 we get the line $2^1\ 2^1$, of the same length. Having got these lines, draw vertical lines from each end of the line $1^1\ 1^1$, to represent the lateral edges of the block.

We now come to point 3. The line represented by

Fig. 22.

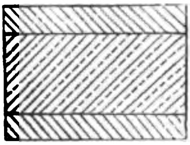


this point would extend from one side of the cube to the other if there were nothing to intercept its course; but since the block forms a part of the cube, and projects from the centre of its face, it will be evident that the line will only extend between the edges of the block and the sides of the cube. Therefore, from point 3, draw lines, commencing at each edge of the block at the points 3^1 3^1, making those lines equal in length to *a b* or *a c*, because that edge of the block marked *f* is farther from the eye than that face of the cube marked *g*, by the distance *a b* or *a c:* then as the line 3^1 3^2, on each side of the block, is equal to *a b* or *a c*, the entire distance 3^2 3^2 will be equal to the length of one of the boundary lines or faces of the cube. From points 3^2, 3^2, let fall vertical lines representing the lateral faces of the cube. Find the elevation of the remaining portions of the blocks projecting from the lateral faces of the cube by drawing lines from points 4 and 5, making such lines equal in length to the thickness of the block, as before described. The lower part of the figure being simply a repetition of the upper part, it only remains to be said that the section of the cube with blocks commences at point 2, and terminates at point 6. Those portions of the blocks from 1 to 2 and from 6 to 7 being in elevation, are not interfered with by the line of section. [See Drawing B, on which will be found the remaining sections.]

ART. 41.—Before we proceed with the next figure, it will be necessary to direct attention to the use of dotted lines, thus: - Lines of this description are employed to indicate the direction of the visual or projecting rays, as before explained: but their principal use is to represent some object which is behind another object. As a familiar illustration, let us suppose that the elevation of a fire-place, with mantel-piece, &c., is required, and that it is also necessary to show the breadth and direction or course of the flue. In this case the flue should be represented by dotted lines, because it cannot be seen. The rule, therefore, is simply this:—*All objects which are visible must be represented by full lines; but those objects which are not visible, and yet require to be represented, must be drawn in dotted lines.* In the case to which we have just referred, the dotted lines supersede to some extent the use of a section, inasmuch as the direction of the flue could not consistently be represented by full lines if the drawing were intended to show an elevation.

Art. 42.—Dotted lines are sometimes employed as section lines in the following manner: — Suppose Fig. 23 to represent the section of a piece of wood in combination with two pieces of iron or other metal. It is necessary in the first place to draw the section lines in opposite directions (Art. 14); and in order to distinguish the wood from the metal, it is the practice of many draughtsmen to make every other section line of the wood a dotted line. The section lines of wood are, however, frequently drawn in color, such as burnt or raw sienna; but in engravings this is impracticable, so that it may be desirable to adopt the above practice as a rule.

Fig. 23.

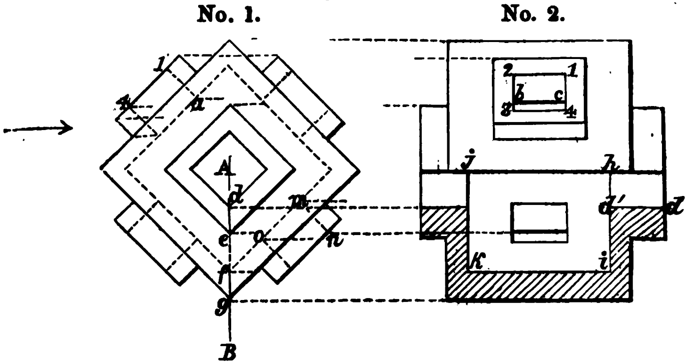


Problem VII.

To find the elevation and sectional elevations of a hollow cube with projecting blocks.

Art. 43.—Fig. 24 represents a box or hollow cube, on each face of which, and in the centre thereof, there is a projecting hollow block or rectangular tube, communicating with the inside of the cube. The extent of the openings and the thickness of wood whereof the object is composed are clearly shown by the dotted lines. It is

Fig. 24.



now required to produce the elevation and sectional elevations, of which there are five,—the lines of section being drawn through the same points as directed for Fig. 20.

Art. 44.—Having got the projected elevation of the cube with blocks, as directed in Art. 38, we have simply to get the elevation of the opening, which is nothing more than an elevation of a square or rectangular tube, making an angle of 45° with the intersecting line, and consequently, when projected, an angle of 45° with the plane of projection (Art. 34). On referring to No. 1 in the above figure, it will be perceived that the *size* of the opening, in the *centre* of the square block, is shown by full lines, because, in this view of the cube, the opening is presented to the eye. Now, on looking at No. 1 in the direction indicated by the arrow, it is evident that a line representing the upper edge of the opening would be seen at point 1, and that its lower edge would be seen at point 4; therefore from point 1 draw the line 2 1, and from point 4 the line 3 4. If the thickness of the wood composing the tube be now set off from each edge of the block, and vertical lines be drawn through those points to represent the sides of the opening, the length of the lines 2 1 and 3 4 will be determined.

There remains another line to be drawn, namely, from the point *a*; for if the edge of the T square be placed upon that point, it will be found to occupy a position between points 1 and 4;—that is to say, if the figure were viewed in the direction indicated, we should see through the opening and into the inside of the cube; and, consequently, we should perceive a line at point *a*, constituting the inner edge of the upper side of the opening. Therefore, from point *a* draw the line *b c;* and the elevation of the upper half of the hollow cube will be complete.

It is desirable to guard the student against a very common error which the writer has frequently had occasion to correct, namely, that of drawing a line from the point in the dotted line 1 *a* where it passes the boundary line of that face of the cube on which the block or tube 1 4 is placed. If the cube and block were distinct, or composed of different materials, such a line would then be correct; but as the block forms part of the cube, it would be incorrect to draw such a line.

ART. 45.—It is a common practice, when space is an object, to show one half of a figure in elevation and the other half in section; but the student is recommended to complete the whole figure as an elevation, and then proceed with the first section, which, for the reason just stated, we have exhibited as the lower half of No. 2, Fig. 24.

ART. 46.—We must now presume that No. 1, Fig. 24, is divided by the line of section A B, and that one-fourth, on the left hand of the figure, is removed. Perhaps it would be more convenient to suppose the line of section to be produced and half the figure removed. If this were the case, we should then have a section showing the inside of the cube now to be explained.

Taking *d*, the highest point in the line of section A B, as the first for illustration, it will be necessary to consider what length of line would be produced if the cube were divided at that point by a plane cutting through the several points from A to B; in other words, what is the distance from point *d* to the next point beyond it?

In the first place, we have got a cubical box, made of wood or other material, of the thickness represented by dotted lines; consequently, if we set off that thickness from each side or lateral face of the cube, and let fall vertical lines, *h*, *i*, *j*, *k*, through those points, we shall then have two lines representing the inner faces of the cube. Now, the line *d* extends from the inner face of the cube to the outer face of the tubular block; therefore, from *d* draw the line *d′ d*, No. 2, and do the same on the opposite side of the cube.

It will be remembered that the points *e* and *g* were obtained when getting out the elevation, so that we have only to notice the point *f*, which represents a line extending across the inside of the cube; therefore, from point *f* draw *k i*, and put in the section lines as shown.

The next question is, what should we see inside the cube if one half were removed? Manifestly the points *m*, *n*, *o* (which have already been described in the shape of the like points, 1, *a*, 4), representing an opening in the centre of the cube, as shown in No. 2.

ART. 47.—It is presumed that during the reading of this book the student is occupied with his pencil in putting down each line as it is described, and that he never makes a line without understanding its meaning. It would be an easy matter indeed to copy a number of mathematical symbols; but such a practice would never make a mathematician: neither would the practice of copying mechanical drawings or the examples given in this work make a mechanical draughtsman; although in course of time he might be able to produce a very elaborate and highly finished piece of work in the form of a copy of another drawing. Our object, however, is to produce something more than a mere copyist; and if the practice here pointed out be perseveringly fol-

termination

lowed, it is confidently hoped that by the time the student has gone through all that the writer intends to lay before him, he will take a higher position than the person who has acquired his knowledge of mechanical drawing by copying or imitation rather than by the study of its principles.

Fig. 25.

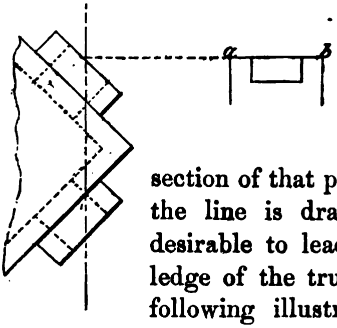
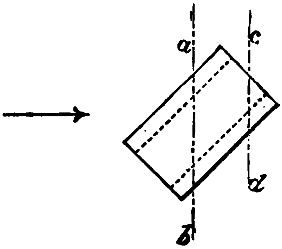


Fig. 26.



Art. 48.—In getting out the sectional elevations of the hollow cube, the teacher will invariably find that the first point in the fourth section is a great stumbling-block; that many of his students will persist in drawing a line, as *a b* (Fig. 25), to represent the section of that part of the tube from which the line is drawn. In such cases it is desirable to lead the student to a knowledge of the true projection by giving the following illustration (Fig. 26), which represents a tube 1 inch square, and 2 inches long, the size of the opening or interior being shown by dotted lines. Required, first, the elevation; secondly, a sectional elevation taken through the line *a b;* and thirdly, a sectional elevation taken through the line *c d.* It will then be found that the line of section at the point alluded to does not extend across the figure, but is represented by two lines. Should any of our readers make the same mistake, let them try the above solution before they refer to Drawing C, on which the fourth section will be found.

Art. 49.—We will now proceed with an explanation of the fifth section (see Fig. 27, Plate 1), first remarking that all the sections have been so arranged that the fifth section, although by far the most difficult, contains nothing more than will be found in one or the other of the preceding sections; as a general rule, therefore, the student is expected to go through the fifth without the aid of his instructor.

Inasmuch as this section forms a key to all the preceding sections, it will be necessary to dwell upon it at considerable length. There are certain portions, however, which may be left to the comprehensive powers of the student, to avoid the use of additional letters of reference.

ART. 50.—It is now presumed that the lower half of the cube is removed, and that the upper or remaining half is viewed in the same direction as before. It will be remembered that points 1, 2, and 3, No. 1, which are represented by lines 1′ 1″, 2′ 2″, 3′ 3″, No. 2, were described when getting out the elevation; we shall, therefore, commence with point 4. If the block were solid, this line would extend across its face, and its length would be equal to one side of the square; but since there is an opening through the block to the inside of the cube, it is evident that a plane passing through point 4 would produce, in the section, two lines, each being in length equal to the thickness of the sides of the tube. Having got the elevation of lines 1, 2, and 3, and determined their lengths (ART. 44), let fall from the end of each line vertical lines, which will represent the lateral faces of the cube and tube, and the inner faces of the tube; the latter being represented by the vertical lines drawn from 3′ 3″. Since the vertical lines from points 2′ 2″, 3′ 3″, represent those sides of the tube which are cut by the line of section at point 4, it is obvious that the projection of point 4 will be 4′, and that 4″ will be the projection of the line beyond point 4 which is on the farther side of the tube from that seen at No. 1.

We now come to point 5. If the figure were a plain solid cube, the line of section at this point would extend across its face (ART. 40); but its course is interrupted [see point 3 in the third section, Fig. 22], and consequently the projection of point 5 will commence at 5′ and terminate at the block; and on the other side of the block the line will extend to 5″.

The next point cut by the line of section is a dotted line, representing the inner face of the cube, marked 6. This line, like that at point 4, would extend across the inside of the cube were it not for the opening, which interferes with its course; consequently the line will commence on the inner face of each side of the cube and terminate at the opening. Therefore, from each lateral face of the cube set off its thickness, and draw vertical lines, *a b*, *c d*; from those lines draw *a* 6″, *c* 6′; and the section at point 6 will be obtained.

Point 7. The line of section at this point projects from the lateral faces of the cube a distance equal to the thickness of the block [see points 4 and 5, Fig. 22]. Through each end of the lines drawn from point 7, marked 7,′ 7,″ draw vertical lines, *e f*, *g h*, representing the ends or lateral faces of the tube; and from point 8 draw the lines *g* 8, *e* 8. It will be perceived, on referring to No. 1, that the part 7 8 of the tube, and also the portion from 4 to 2, as well as the face of the cube from 5 to 1, are not interfered with by the line of section, and are therefore shown in elevation.

The 9th point represents a line extending from the outside of the tube to the inside of the cube, as described with reference to point *a*, Fig. 24, and represented in No. 2 of the present figure by the line 9′ 9″ Again, point 10 in the section is the same as 9; point 11 is the same as 7; 12 is the same as 6; and 13 is the same as 5. The length of the lines from point 13 is determined by letting fall vertical lines from 2′ 2″, to represent the outside of the lower tube; and if similar lines be drawn from 3″ 3′, representing the inside of the tube, the extent of those lines will be determined by drawing a line from point 14, which is the same as 4. We may now put in the section lines, which commence at point 4 and terminate at point 9, and commence again at point 10, terminating at point 14, as shown in No. 2.

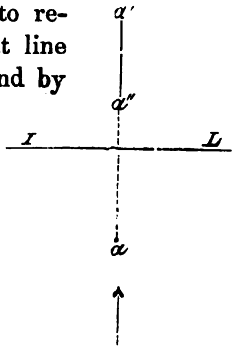
We have yet to consider what will be seen inside the cube when the lower half is removed. In the first place we shall see the lines indicated by the points *k, l, m, n, o.* These lines, it will be remembered, represent the angular edges of the apertures, and are therefore of the same length as the line 3″ 3′; and since the openings are in the middle of the cube, the projected lines drawn from those points will be immediately below the line 3″ 3′. Therefore, from points *k, l, m,* draw lines; and produce the vertical lines from points 3″ 3′ to meet the line from *m,* in *m″ m′.* Draw similar lines from *n* and *o;* and produce the vertical lines representing the inside of the opening to meet the line from *n* in *n″ n′.* From the point *r,* or *s,* draw a line across the entire figure, as *r′, s′, r″.* This line, it will be observed, is the representative of three lines,—namely, the line from point *r* to the point beyond it (that is, from the outside of the tube to the inside of the cube), the corresponding line on the other side of the figure, and the line from point *s* to the point beyond, which extends across the inside of the cube. This is only true when the diagonals of the cube are vertical and horizontal, in which case the points or lines *r* and *s* coincide; but in any other position they would be projected into three distinct lines.

CHAPTER IV.

PROJECTION FROM THE LOWER TO THE UPPER PLANE.

ART. 51.—It has been shown that the two planes of projection are at right angles to each other (ART. 3). Consequently, if any given point, *a*, in the horizontal plane be supposed to represent a line, the projection of that line in the vertical plane would be found by drawing from the given point a line at right angles to the intersecting line, as *a a'*, and making *a' a''* equal in length to the supposed length of the line represented by the point *a*. In this case *a* represents a plan of a line at right angles to the plane of projection, and *a' a''* the elevation of that line, which is parallel to the plane of projection.

Fig. 28.



ART. 52.—If this book be now inverted or turned upside down, the reader will see that the foregoing proposition is just the converse of that given in ART. 5; for, in that position of the book, *a* becomes the elevation of a point or line, and *a' a''* the plan of that line. It is proposed, however, to give a few examples of projection from the lower to the upper plane, in order that the student may become familiar with some objects which in Chapter VI. will be projected upon an inclined plane. It is also of importance to remember that when an *elevation* only of an object is required, the principle contained in the following propositions will in all cases apply; but

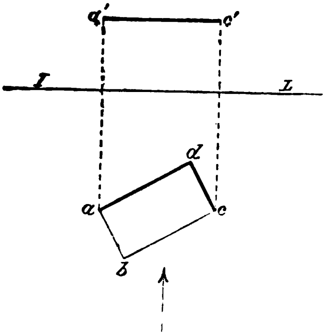
when the projection of an object is required on the *inclined plane*, the case will be very different, as will be hereafter explained.

Problem VIII.

The plan of a rectangular plate being given, to find the elevation.

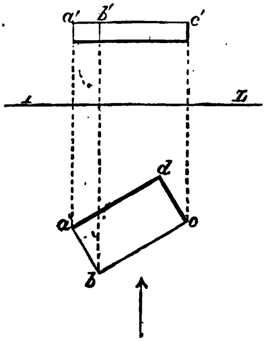
Art. 53.—If the lower part of Fig. 29 be supposed to represent a plan of a rectangular plate or sheet, *a b c d*, of paper, tin, or other like material, parallel to the lower plane, its elevation will be represented by a right line drawn between the vertical lines projected from the points *a* and *c*. Then will *a′ c′* be an *edge view* or elevation of the plate *a b c d*, which is parallel to the lower plane, and at right angles to the upper plane (Art. 35).

Fig. 29.



Art. 54.—If we suppose the rectangular figure to be the plan of a block of wood or stone, instead of a plate, the points *a, b, c, d,* will then represent lines, each being in length equal to the thickness of the block. In obtaining the elevation of the block, as at Fig. 30, the plan is supposed to be viewed in the direction indicated by the arrow. In this case the points *a, b, c*, only are seen; and their projection is obtained as already described for the projection of a single point or line (Art. 51).

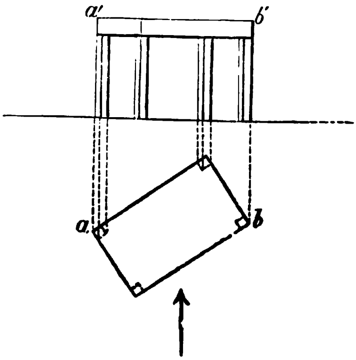
Fig. 30.



ART. 55.—Now the elevation of point *b* is *b′*, that of *c*, *c′*, and of *a*, *a′*. Again, point *b′* is nearer to the eye of the observer than point *a′* or *c′*; and *b* is the plan of *b′*. *Therefore all those points which are nearest to the eye in the elevation will in the plan be farthest from the intersecting line; and, conversely, all those points which are farthest from the eye in the elevation will in the plan be nearest to the intersecting line.*

ART. 56.—The writer has repeatedly been asked what distance the elevation should be above the intersecting line. In reply, it may be stated that the height of any figure above the intersecting line depends entirely on circumstances: perhaps it would be more consistent to place the elevation of Fig. 30 upon the intersecting line. As a general rule, however, it may be said that *whatever distance the elevation is above the intersecting line, the plan may be looked upon as being the same distance above the horizontal plane.* In many cases, however, the distance above the intersecting line is a matter of convenience. As an illustration of the rule, we may suppose the lower part of Fig. 31 to represent the plan of a table, in which the positions of the legs are indicated by dotted lines (ART. 41). Now the table is supposed to stand upon the horizontal plane; and since the intersecting line is sometimes called the ground line, and is intended to represent *an elevation of the horizontal plane,* it follows that the legs of the table, in the vertical plane, should stand upon the ground line: hence the rule that the height of *a′ b′* above the intersecting line represents the distance of the upper surface of the table, *a b*, from its plane of projection.

Fig. 31.



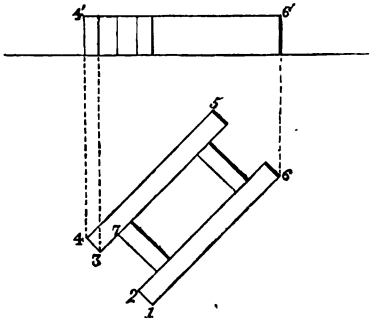
The positions of the legs in the elevation are obtained as shown by the dotted lines, which represent the projecting rays.

PROBLEM IX.

Given the plan of a rectangular frame, to find the elevation.

ART. 57.—Let 1 4 5 6, Fig. 32, represent the plan of a wooden frame of any dimensions, except that the breadth of material of which it is made is equal to twice the thickness; that is, the length of the lines represented by points 1, 2, 3, 4, &c., is twice the thickness from 1 to 2, or from 3 to 4. Required the elevation.

Fig. 32.



In this figure the projection of the lines from points 1, 2, 3, 4, 7, 6, only are required; and since the lines are all of the same length, their elevation will be bounded by the horizontal line 4′ 6′, representing the upper face of the figure, and by the ground line on which the object is supposed to rest.

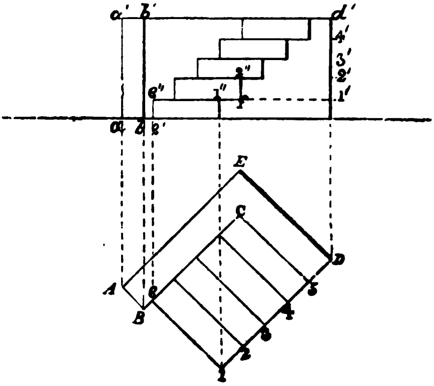
PROBLEM X.

The plan of a flight of steps being given, to find the elevation.

ART. 58.—Let Fig. 33 represent the plan of a flight of steps, on the left of which there is a wall projecting a little in front of the bottom or first step. The height or rise of each step is equal to two-thirds of the breadth of the horizontal face. It is required to find the elevation.

The vertical corners of the steps are represented by points 1, 2, 3, 4, 5, each being in length equal to two-thirds of the breadth of the horizontal face, or distance from 1 to 2. The upper face of the wall, A B C, is level with the top step; consequently the length of lines represented by points A, B, D, will be equal to the rise of the five steps, or two-thirds of the distance from 1 to D. Therefore, from A, B, D, draw $a\ a'$, $b\ b'$, $d\ d'$, at right angles to the intersecting line; and having determined the length of these lines, draw $a'\ d'$ parallel to the ground line: then will $a'\ d'$ represent the upper face of the wall and the top step. Divide the vertical line d' into five equal parts, $1'$, $2'$, $3'$, &c.: then will $1'$, $2'$, $3'$, &c., represent the vertical heights or distances of the points 1, 2, 3, &c., from the lower plane. If, therefore, vertical lines be drawn from points 1, 2, 3, &c., in the lower plane, and horizontal lines from the points $1'$, $2'$, $3'$, &c., in the vertical plane, the intersection of these lines will give the elevations of the corners of the steps, as at points $1''$, $2''$. The upper faces of the steps are represented by lines drawn through the aforesaid intersections, parallel to I L; and the length of each step is determined by drawing vertical lines from those points where the steps join the face of the wall, as shown by line $e'\ e''$, drawn from e, and by line $f\ 2''$, drawn from point 2.

Fig. 33.



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CHAPTER V.

ON SHADOW OR SHADE LINES.

ART. 59.—We purpose in this Chapter, which will be as brief as the nature of the subject will allow, to explain the use of what is called a shade or shadow line.

In the execution of mechanical and architectural outline drawings, it is necessary to make use of different grades or thicknesses of lines, which are called *fine*, *medium*, and *shadow* or *shade* lines. Although these lines may be looked upon as embellishments to the drawing, their function is of a much more important nature, inasmuch as the incorrect use of a shadow line will produce an effect the very reverse of that which the draughtsman intended. It is, therefore, of great importance that those persons who are engaged in this description of drawing should adopt a uniform rule with respect to the direction in which the light is supposed to fall upon the picture. We are induced to call attention to this, from the fact that we have sometimes met with drawings having the light introduced in the opposite direction to that in which, as a rule, it is generally supposed to fall. In order that the proper direction in which the light should be admitted may be clearly understood, the student is requested to place his drawing board before him in a vertical position. If he can now imagine a ray or rays of light coming over his *left* shoulder so as to strike the planes of the drawing at an apparent angle of 45° with the ***intersecting line,*** he will have an idea of the proper direction of the light. *This is the rule.* In the above-mentioned departure from the rule, the light is supposed to come over the right shoulder at the same angle; and we beg to caution the student against adopting this practice.

ART. 60.—Let *a b d c*, No. 1, Plate II., represent the two planes of projection, divided by I L, the intersecting line. Upon the upper and lower planes draw *e h l g*, to represent the elevation, and *i l′ h′ e′*, the plan of a cube. From *l* and *l′*, through *e* and *e′*, draw lines *l e n*, *l′ e′ n′*: then will *n e l* represent the direction of the ray or rays of light falling on the vertical plane, and *n′ e′ l′* the direction of the rays of light which fall on the horizontal plane. We will now suppose the two planes to be bent (or turned upon I L as a hinge joint) at right angles to each other. If, in this position, the two planes, with their cubes, were viewed in the direction of the arrow, they would appear as shown at No. 2, in which it will be seen that the rays of light falling on the two planes are parallel to each other. It will also be observed that the *apparent* angle which those rays *n e l*, *n′ e′ l′*, make with the planes of projection is 45°; but the *real* angle is 35° 16′. The mode of proving this will be given in a subsequent Chapter; for our present purpose it will be sufficient to state that the rays of light are represented in the upper and lower planes by lines *drawn* at an angle of 45° with the intersecting line.

ART. 61.—We now come to an explanation of the object of making a distinction in the thickness of lines when executing outline drawings. As general rules we may here state:—

First.—That all lines forming those angles or boundaries of surfaces upon which the light falls direct, and which do not cast a shadow upon any other surface or object, must be *fine lines.*

Secondly.—All lines forming those angles or boundaries of surfaces upon which the light does not fall direct, and from which a shadow is cast upon some other surface or object, must be *thick lines*, commonly called shadow or shade lines.

Cylindrical, conical, and spherical surfaces are exceptions to these rules, as will be hereafter noticed.

Art. 62.—If the light be supposed to fall on the cube *e h l g*, No. 1, as already explained, three of its faces will be illumined and three in shade. It is, therefore, evident, from the direction of the light, that the boundary lines or angles of the cube *e h*, *e g* will follow the first rule; but since the lines *l g*, *l h* cast a shadow on the plane *a b* L I, of the form shown at *o p q*, they will follow the second rule.

In like manner, since the light falls upon the lower plane in the direction *n′ e′ l′*, the boundary lines *l′ i*, *l′ h′* will cast the shadow *r s t*: therefore *l′ i*, *l′ h′* must be shade lines; and, for the reasons above given, *e′ i*, *e′ h′* must be fine lines. Having explained the direction in which the light falls upon the drawing, and the position of the two planes, it is necessary to observe that English draughtsmen generally project the shadow on the lower plane in the same direction as the shadow in the upper plane, as shown by dotted lines, *u v w*; in which case *h′ e′* and *h′ l′* would be shade lines. But as this practice can only be defended on the supposition that all plans and elevations of structures are in the same plane, as indicated by the dotted line *x y*, No. 2, we prefer to adopt the practice of the French, who recognise the existence of the two planes, in accordance with the principles of orthographic projection.

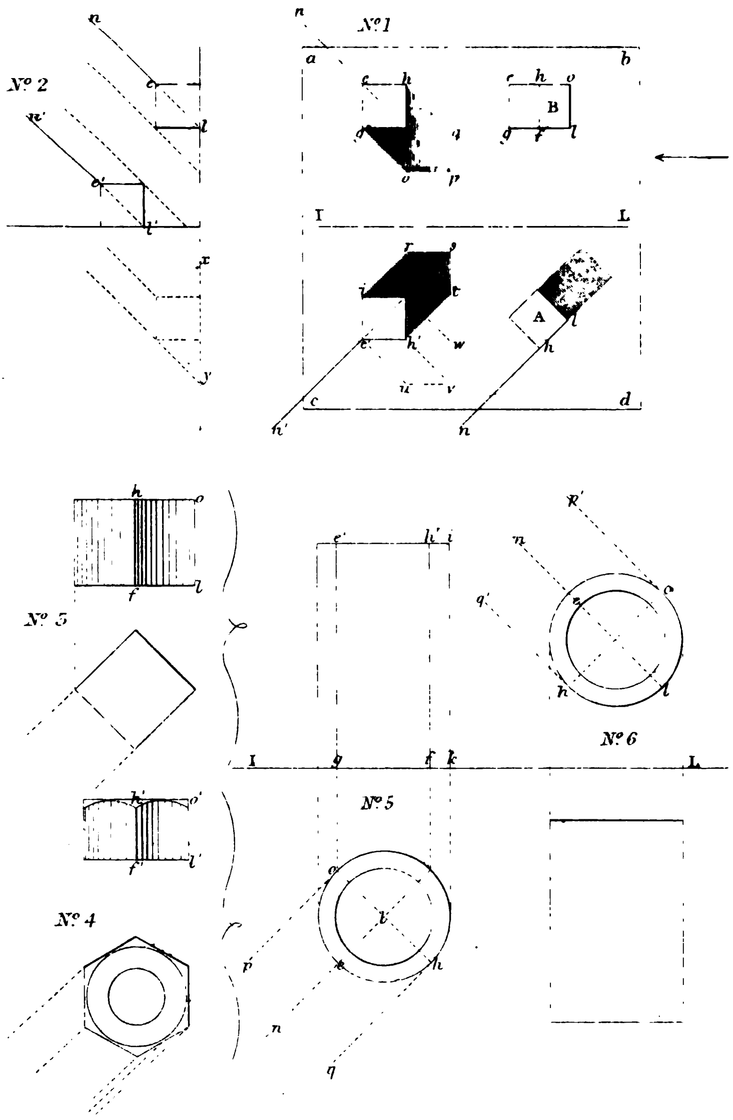
Art. 63.—We will now suppose the cube *e h l g*, No. 1, to be turned as represented in elevation at B, and in plan at A, so as to present two of its faces to view; in which case it will be evident that the vertical boundary line *h f*, at some portion of a revolution, would cease to be a shadow line and become a fine line. Let the cube A be moved into such a position that the face B is parallel to or in the same plane as the rays of light, as indicated by *n h l* in the plan A. In this position of the cube it is evident that the light would fall direct on the face *e h f g*, and that the face B would be in the shade. It now becomes a question as to which of the vertical boundary lines *o l* or *h f* should be the shadow line. A very common practice is to make *o l* a shadow or thick line, and *h f* a fine line; we prefer, however, in cases where *o l* is a definite shadow line, to make *h f* a medium line, with a view to distinguish an angular surface from a flat surface upon which a right line might be drawn. At the same time it is questionable whether *h f* should not be the shadow line and *o l* a fine line. If we refer to No. 3, which represents a plan and shaded elevation of a cube, it will be seen that the darkest part of the shade commences at the line *h f* and terminates with a reflected light as it approaches *o l*. The practice of shading hexagonal nuts, although the light falls upon the face at an acute angle, is the same as when shading the cube, notwithstanding that *o′ l′*, No. 4, becomes, according to the rule, a definite shadow line. Again, if the nut were turned so as to present a little more of its darkened face to the light, the shadow would lose much of its intensity, and ultimately *h′ f′* would become a fine line.

It is for the above reasons that we recommend for angular surfaces *either a definite shadow line with two fine lines*, or *a shadow and a medium shadow line with one fine line*. The thickness of the medium line and the angle at which it should become a fine line must be left to the judgment of the draughtsman, especially for objects projected upon an inclined plane [see Drawings D and E], for which it is difficult to lay down rules. It will be observed, however, that these projections, like those in isometrical perspective, exhibit a plan and elevation in one view; and as such delineations are sufficiently clear without shadow lines, their application may be considered more as an embellishment than as a matter of utility. Moreover, if the student attempt to adhere strictly to any rule, he will find the greater portion of his work composed of either medium or shade lines, and the question of their discontinuance a difficult one to decide. An illustration of this will be seen on referring to Drawing E. It must, therefore, be understood that the above rules are intended to apply to orthographic representations of objects, and not to isometrical delineations or projections upon the inclined plane.

ART. 64.—With regard to cylindrical surfaces, it is not only inelegant but improper to apply either a medium or shade line. If the reader will turn to Drawings K and L, he will there see the manner of producing the effect of a cylindrical surface, which is greatly enhanced by leaving a reflected light on the right hand of the darkest part of the cylinder. This is also the case with cones and spheres, which must be drawn without medium or shadow lines.

Let No. 5, Plate 2, represent the plan and elevation of a cylinder. From *l*, the centre of the plan, draw *l e n* at an angle of 45° with the intersecting line. Draw *o l h* at right angles to *l e*, cutting the circumference of the cylinder in points *o* and *h*. From the points of intersection *e* and *h*, draw vertical lines, e' *g*, h' *f*. Then will e' *g* be that part of the cylinder upon which the light falls direct, and h' *f* that portion of the cylinder's surface which would cast a shadow upon the plane of projection. Consequently e' *g* will be the lightest part of the cylinder, and h' *f* the darkest part. It will be evident, on examining the direction of the rays of light *o p*, *h q*, which are drawn parallel to *l e n*, that it would be highly improper to apply the second rule to the boundary line *i k*, although this is too frequently done in the beautiful illustrations of our works. Apart from such a direct innovation of the principles of shading, if this practice were discontinued, cylindrical surfaces in an outline drawing would then be readily distinguished from flat surfaces; in other words, a round bar of iron would never be taken for a flat or square bar—an advan-

PLATE 2



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tage which in itself is sufficient to show the necessity of discontinuing shadow lines on cylindrical surfaces. It is correct, however, to shade-line the end of a cylinder; and this is effected in the following manner:—

ART. 65.—Let No. 6 represent the end elevation of a cylinder, and *l e n*, *o p′*, *h q′*, the rays of light. Draw *h o* at right angles to *l e n*. Then will *h o* determine the extent of the shade line for the interior and exterior of the cylinder. For the interior, *e* will be the darkest part of the shade line, which must be gradually reduced in thickness as it approaches the line *h o*. For the exterior, *l* will be the darkest point, the line being gradually reduced in thickness towards *h* and *o*, where it joins that part of the circle or end of the cylinder upon which the light falls direct. It will be seen that the plan, No. 5, is shaded in like manner. The mode of producing these graduating lines for large circles is by opening or unscrewing the bow pen and gradually closing it; and for the small circles, by commencing at the darkest part and going over the line a sufficient number of times to obtain the desired effect.

As an illustration of the result produced by admitting the light over the right shoulder, we may direct attention to the plan of Fig. 9, Drawing A, in which, if the shade lines were drawn on the opposite side, that part which is intended to represent a groove or recess would, by a person who had been accustomed to the rule, be taken for a projection, or part of the object standing above the rest.

Lastly.—On referring to the first elevation of the cube on Drawing B, it will be seen that those faces which are parallel to the rays of light are not shadow-lined

CHAPTER VI.

PROJECTION UPON THE INCLINED PLANE.

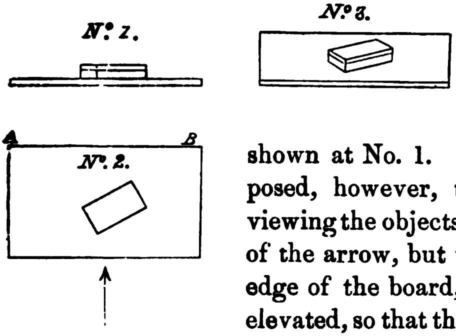
ART. 66.—As the surest means of imparting a knowledge of the principles which govern the projection of objects upon the inclined plane, it is proposed to carry the student step by step through the projection of those figures with which he has become familiar. Although this description of drawing seldom finds its way into the workshop, because the simplest and most accurate mode of representing anything in process of manufacture is by plans, elevations, and sections, yet without a knowledge of this interesting portion of our subject, it would be impossible to get the projection of an object which is inclined to the plane of its projection. The purport of this Chapter is, therefore, to enable the merest tyro to find the elevation or plan of an object, however complex, when that object makes an angle with the vertical or horizontal plane greater or less than a right angle.

ART. 67.—It is necessary to explain, with regard to the following examples, that in all cases the original object for projection will be given upon the *horizontal plane*, and will, therefore, be called a plan of the object to be represented or projected upon the *inclined plane.*

ART. 68.—In the preceding part of this work we have given a number of examples of projection from the upper to the lower plane, projection in the upper plane, and, lastly, projection from the lower to the upper plane. In all these examples it has been understood that the two planes of projection are at right angles to each other, the upper plane being vertical and the lower plane horizontal. It is now proposed to explain the projection of objects upon planes which are not at right angles to each other: that is to say, the upper plane shall be an *inclined* plane, making an angle with the vertical or horizontal plane greater than a right angle.

Art. 69.—As a familiar illustration of the inclined plane, let the student place his drawing board horizontally upon the table, and his box of instruments upon it, as represented at No. 2, Fig. 34. If the board and box were viewed in the direction indicated by the arrow, we should get an elevation, as shown at No. 1. Let it be supposed, however, that he is still viewing the objects in the direction of the arrow, but that the farther edge of the board, A B, has been elevated, so that the face may form any given angle with the visual rays. In this case the objects would assume something of the appearance exhibited at No. 3, depending upon the inclination of the board. We have, therefore, at No. 3, an illustration of the projection of a box upon an inclined plane.

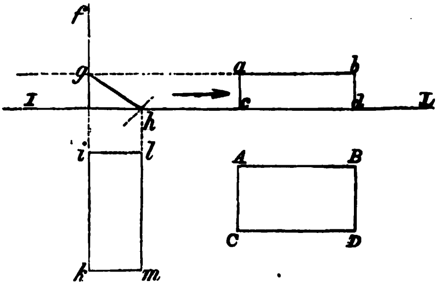
Fig. 34.



Art. 70.—Before we proceed with an exposition of the principles which govern the delineation of objects when seen at any given angle, it will be desirable to examine more minutely the relative positions of the vertical, inclined, and horizontal planes, and also the relation of the three to each other.

Let A B D C, Fig. 35, represent a plan of the drawing board given at Fig. 34; and *a b d c*, its elevation when inclined to the vertical plane. It is now required to get an edge view or elevation of the board when looking at it in the direction indicated by the arrow. In other words, given A B D C, the plan, and *a b d c*, the elevation of a plane, to find the angle which that plane makes with the vertical plane.

Fig. 35.



Produce the line *b a* indefinitely; from any point, *g*, in such line, with a radius equal to A C or B D, describe an arc cutting the ground line in the point *h*; join *g h*: then will *g h* be equal to the breadth of the board, and therefore represent an end elevation or view of the board as if seen in the direction indicated. Through *g* draw *g f* perpendicularly to I L: then will *g f* represent an edge view of the vertical plane, and *f g h* the angle which the board makes with that plane.

ART. 71.—Again, produce *f g* to *i k*, making *i k* equal to A B; from *h* draw *l m* parallel to *i k*; join *i l* and *k m*: then will *i k m l* be a plan of the inclined plane *g h*. The plane *i k m l* will therefore make an angle with the horizontal plane equal to the angle *g h* I.

If the reader has thoroughly succeeded in realising the relative positions of the drawing board which has been used to illustrate the planes of projection in this Chapter, a knowledge of the projection of objects upon those planes will be readily acquired, provided attention be paid to the following theorems.

THEOREMS.

ART. 72.—1st. The projection of any given point in either the upper or lower plane will always be in a line drawn from the original point in that plane parallel to the intersecting line.

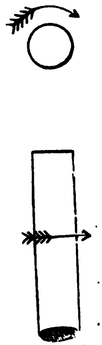
2nd. The projection of a point from the upper to the lower plane, or from the lower to the upper plane, will always be in a line drawn from the original point at right angles to the intersecting line.

3rd. When one or more original points are given in the lower plane, and corresponding points to the originals in *any part* of the upper plane, the projection of such points will be found by drawing lines from the original points at right angles to the intersecting line, and from the corresponding points parallel to the intersecting line,—the projection of the original and corresponding points being where the lines so drawn cut each other in the upper plane. The converse of this is the case when the original points are given in the upper plane and the corresponding points in the lower plane.

4th. All lines which are parallel to the vertical plane will, in the lower plane, be parallel to the intersecting line

5th. All those points which are nearest to the eye in the upper plane will be farthest from the intersecting line in the lower plane. Conversely, all those points which are farthest from the eye in the upper plane will be nearest the intersecting line in the lower plane. [Read Art. 20.]

Fig. 36.

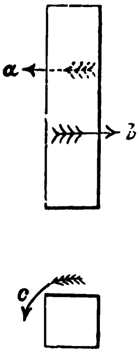


Note.—In the succeeding pages the arrow drawn thus ——> will denote the direction in which an object is supposed to be seen, as heretofore; and this arrow »»»——> will be used to point out the direction in which any figure is supposed to move.

Art. 73.—*Example* 1. Let Fig. 36 represent the end elevation and plan of a shaft or cylinder turning upon its axis as indicated by the arrows. It will be evident, more especially from the upper figure, that the direction of motion here pointed out is the same as that of the hands of a watch, and the apparent motion of the sun. This motion is therefore said to be right-handed.

Art. 74.—*Example* 2. Suppose Fig. 37 to represent the elevation and plan of a square prism. We can imagine the prism to turn upon its axis* in one direction or another. In this case the motion is supposed to be from right to left; and it is therefore called left-handed. Although the arrows *a* and *b* point in opposite directions they represent the same direction of motion; because *a* is drawn partly in dotted lines, and is therefore a correct elevation of the arrow *c* (Art. 41).

Fig 37.



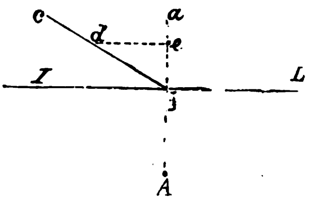
We shall now proceed with the projection of points, lines, plane figures, and geometrical solids upon the inclined plane

Problem XI.

Given the intersecting line and angle of the planes, to find the projection of a point.

Art. 75.—Let A, Fig. 38, be the given point. Draw A *b a* perpendicular to I L, the intersecting line; and make the angle *c b* I equal to the angle of the two planes. From *b*, with the distance *b* A, cut *b c* in *d*; draw *d e* parallel to the intersecting line, meeting *a b* in *e*; and *e* is the projection of the point A.

Fig. 38.



It is not necessary that the line *a b* A should pass through the foot or base of the inclined plane, as will appear from the following problem.

---

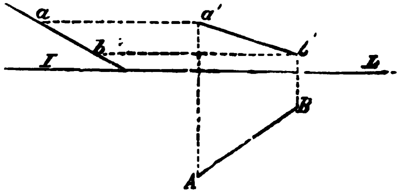
* The axis is a line, real or imaginary, that passes through anything on which it may revolve; or, in other words, it is the centre of motion.

PROBLEM XII.

Given the intersecting line and inclination of the plane, to find the projection of a given right line.

ART. 76.—Let A B, Fig. 39, be the given right line, and *a b*, the inclination of the planes. Find the projection of the points A B (Prob. XI.); join *a′ b′*; and *a′ b′* is the projection of the given right line A B.

Fig. 39.



PROBLEM XIII.

Given the intersecting line and inclination of the plane, to find the projection of a triangle.

ART. 77.—Let A C B, Fig. 40, be the given triangle, and *a c* the inclination of the plane

Fig. 40.

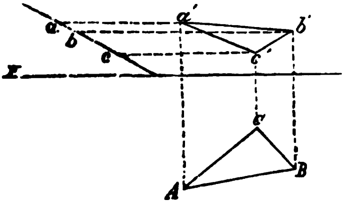
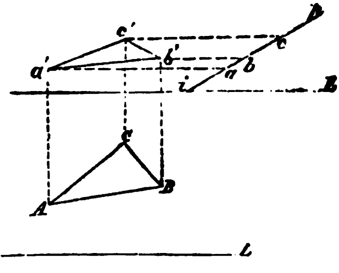


Fig. 41



Find the position of points A, B, C, upon the inclined plane, *by measuring the distances from the intersecting line;* find also the projection of the points *a′*, *b′*, *c′*, and join them by right lines: then will $a' c' b'$ be the projection of the triangle A C B.

ART. 78.—It should be understood that the problems relating to Figs. 38, 39, and 40, have been given more as elementary illustrations of the principles which govern the projection of objects on the inclined plane than as rules to be followed. Moreover, since problems XI. and XII. have been taken from an excellent work on Projection by Mr. Peter Nicholson, to whom we are indebted for some of the figures in this treatise, it is desirable to remark that they are given in the work to which we allude as projections upon the lower plane; whereas, we have given them as projections from the lower to the upper plane; and in the subsequent Chapters we shall have to consider them as projections from the lower to the upper plane, and from the upper to the lower plane. There will also be this difference in the mode of treatment, that, instead of measuring from the intersecting line the distances of the several points, as A, B, C, *such distances will be measured from some line below the original figure.* This line being employed in all our future problems, will be marked with the letters D L, and called the datum line, inasmuch as it is a line from which all lateral measurements for the elevation of the original object upon the inclined plane will be taken.

We now propose to give the projection of the last figure in accordance with this mode of treatment.

ART. 79.—Let A B C, Fig. 41, be the plan of a triangle, as shown at Fig. 40. At any given distance from the original figure draw D L parallel to the intersecting line. Make $i\,p$ equal to the inclination of the plane on which the object is to be projected. Measure the distance of points A B and C from *the datum line,* and set them off respectively from i on the line $i\,p$, in points a, b, c. Find the projection of the points a, b, c, according to Theorem 3. Then will $a' c' b'$ be the projection of the original figure A C B.

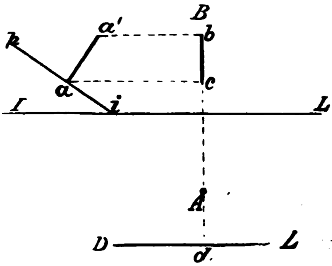
ART. 80.—The difference betwixt measuring from the intersecting line and the datum line would be more apparent, perhaps, if the object represented one of the tiles of a tesselated pavement, with a device on its upper surface; for in that case the device would not be seen in the projection, Fig. 40, inasmuch as the object is turned upside down. Whereas, by measuring from the datum line, the same surface is presented in both figures—the effect being precisely that which we have illustrated with the drawing board. It will also be observed that this mode of proceeding is strictly in accordance with Theorem 5 and the true principles of orthographic projection.

PROBLEM XIV.

Required the projection of a given right line, which is represented in the lower plane by a point.

ART. 81.—If a point, as A, Fig. 42, be made to represent a right line, such line must be at right angles to the plane of projection. Therefore, having found the position of point *a* on the inclined plane (by making *i a* equal to A *d*, the distance of A from D L), draw *a a′* at right angles to *i p*, its plane of projection; and make *a a′* equal to the supposed length of line represented by point A. From A draw A B perpendicular to I L, and from *a′ a* draw *a′ b*, *a c*, parallel to the intersecting line. Then will *b c* be the projection of the point or line A.

Fig. 42.



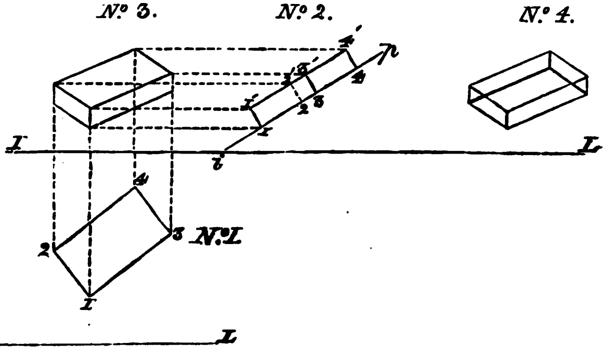
ART. 82.—*Q.* At what distance should the datum line be from the original point or figure?

Ans.—The distance is immaterial, inasmuch as the datum line is simply employed as a means of getting the relative position of the points and lines of which the original figure is composed. By increasing or diminishing the distance of the datum line, the projection of the figure will be farther from or nearer to the intersecting line; all other conditions remain the same.

PROBLEM XV.

Required the projection of a rectangular block, two inches long, one inch wide, and half an inch thick, resting on a plane which makes any given angle with the intersecting line.

Fig. 43.



ART. 83.—Find the position of points 1, 2, 3, 4, upon the inclined plane, as directed in ART. 72. At right angles to that plane, and from points 1, 2, 3, 4, draw lines $1\,1'$, $2\,2'$, &c., making them equal in length to the thickness of the object. Through the extremities of these lines draw $1'\,4'$, parallel to *i p*. Then will $1'\,4'$ represent the upper face of the rectangular block; and since there are four points in that face, corresponding to points 1, 2, 3, 4, in the plan, their projection will be found as directed in

Theorem 3. The points 1, 2, 3, 4, in the lower face of the figure, will be found in like manner, as shown by the projecting rays. If the student will once more place his box of instruments upon his drawing board, and look at it as directed in Art. 69, he will perceive the reason for two lines only being drawn to represent the lower face. Although it would be proper to represent those parts which are not seen by a fine dotted line, it is better in some cases to omit them; for this reason, that where there is a disposition to show all the lines, the writer has not unfrequently seen those which ought to be left out drawn particularly full. The consequence is that the figure commences a display of geometrical gymnastics which somewhat takes the student by surprise. This effect, which was alluded to in the first Chapter, will be very marked in some of the figures in this; as an illustration we may direct attention to No. 4, in which, if looked at steadfastly for a few seconds, the reader may with difficulty recognise No. 3, but it will immediately assume an inverted appearance.

Art. 84.—It is well known that one of the greatest drawbacks to a steady perusal of works of this nature is the unavoidable and frequent use of numbers and letters of reference; the desirableness of suppressing them as much as possible will therefore be readily acknowledged; but in order that at the same time we may be understood, it will be necessary to give some explanation of the phraseology which will be used in this Chapter.

In commencing the projection of any object, the first figure to be drawn is the plan (as No. 1, Fig. 43). This will consequently be called the original figure, because it is the initial figure, from which all horizontal dimensions only can be taken.

Again, No. 2, Fig. 43, will be called the elevation, because it is an elevation of the plan upon the inclined plane; and it is from this figure that all vertical dimensions are or may be taken.

No. 3, Fig. 43, being obtained by projecting vertical lines from the plan, and horizontal lines from the elevation, we shall call this figure the projection of No. 1, or the *original* figure, upon the inclined plane.

ART. 85.—The several points in the plan will generally be denoted by numbers instead of letters, and in such manner that they will run consecutively from the base of the inclined plane. This will be effected by calling that point nearest the datum line 1, the next in succession 2, and so on with the rest. By adopting this system the references will be simple and easy, inasmuch as 3 will always be the next number to 4, &c.

Each line in the elevation will be indicated by one number only. Thus, let it be required to find the projection of the *upper* and *lower* ends of line 2. This being done, and the points joined by a right line, we have then got the projection of *line* 2 in the elevation, or of *point* 2 in the plan. Moreover, each number may be employed to represent all the points and lines in the same plane: an exemplification of this will be seen in the following problem.

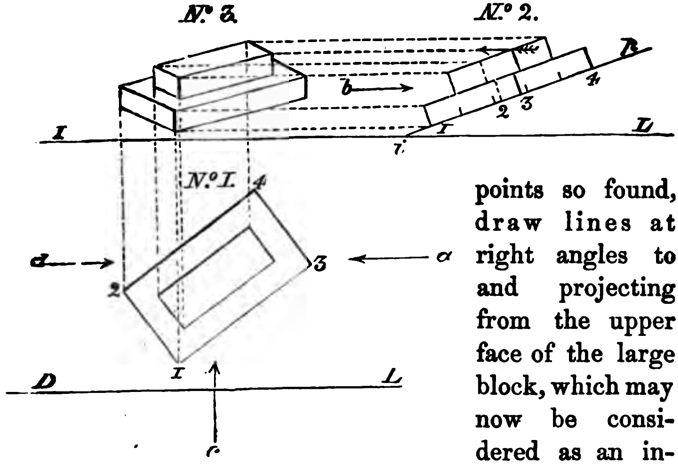
PROBLEM XVI.

Given the intersecting line and inclination of the plane, to find the projection of two rectangular blocks.

ART. 86.—Let 1 2 3 4, Fig. 44, represent a plan of the two blocks, placed one upon the other somewhat in the form of steps, the lines forming the upper block being parallel to those of the lower block. If the thickness of each be the same, the eight points or corners of the two blocks, in plan, will represent so many lines of equal length.

Find the position of points 1, 2, 3, 4, upon the inclined plane; and complete the *elevation* of the lower block as in Prob. XV. Measure the distances of the corresponding points in the smaller block from the datum line, and set them off in like manner on the inclined plane, as shown by small points or dots. Through each of the points so found, draw lines at right angles to and projecting from the upper face of the large block, which may now be considered as an inclined plane, whereon the upper or small block is resting. Now, since the numbers 1, 2, 3, 4, run consecutively, we anticipate there will be little difficulty in ascertaining which is line 1, 2, or 3, in the upper block, and which of the points in the plan will correspond to those lines, since they are directly opposite to their respective numbers. Find the projection of the first four points in the upper face of the small block, as explained in Theor. 3, and join those points by right lines. In other words, find by Prob. XV. the projection of the upper block, as supposed to be resting on the lower block, which forms an inclined plane. The projection of the lower block will simply be a repetition of the operations required for the upper. If this system, which enables us to represent sixteen or more points by four numbers, be practised, we may safely predict a saving of time, with greater simplicity.

Fig. 44.



Art. 87.—The intimate relation which one problem

bears to another throughout this work, renders it of the utmost importance that every part of our subject should be understood as we go on. With this conviction we venture upon a further explanation of the relative position of the three last figures, which it will be important to remember.

In the first place, No. 2 is an elevation of No. 1, or the appearance that figure would present if viewed in the direction of the arrow *a:* hence the reason for representing two of the vertical corners by dotted lines (see lines 2, No. 2),—the visible lines being 1, 3, 4, in both blocks. Again, No. 3 exhibits the same appearance that No. 2 would present if viewed in the direction indicated by the arrow *b:* that is to say, if the plane with the object upon it were turned one-fourth of a revolution, upon *i* as a centre, in the direction of the arrow (namely, left-handed, —ART. 74), so that every point in the figure would describe a plane at right angles to the vertical plane, the appearance of No. 2 would be exactly like that of No. 3. It has also been explained that the same appearance would be exhibited by No. 1, if the corner 4 were elevated to the same extent as No. 2, and viewed in the direction of the arrow *c* (ART. 69). If any proof were wanting of the absolute correctness of the principles enunciated and the propriety of measuring from the datum line when getting the elevation on the inclined plane, we might gradually elevate that plane so as to make an angle of 90° with the intersecting line; in which case the projection would be a fac-simile of the plan No. 1.

ART. 88.—It is also necessary to explain, that when the inclined plane is drawn on the left hand of the figure, as shown at Figs. 38, 39, and 40, the elevation upon that plane will be a view of the plan as if seen in the direction of the arrow *d.* This may be proved by placing a block of wood or box of instruments, with the corners numbered as in Fig. 43, upon a board, inclined first on the right, then on the left and afterwards turned as hereinbefore described, when it will be observed that the numbers will in both cases coincide with or be directly over those in the plan. Therefore, *when the inclined plane is on the right, the elevation will be a representation of the plan as seen in the direction of the arrow a; but when the inclined plane is on the left, the elevation will represent a view of the plan in the direction of the arrow d.* As a general rule, however, we shall place the inclined plane on the right. It will be important, therefore, to remember in what direction the plan is seen when getting projections from the elevation hereafter to be explained.

PROBLEM XVII.

Required the projection of a rectangular frame upon a plane which makes any given angle with the intersecting line.

ART. 89.—Let 1 2 3 4, No. 1, Plate III., represent the plan of a rectangular frame, consisting of two side pieces, 1 3, and 2 4, connected by two cross-bars, as before described. Required its projection in the upper plane,—*p i* L being the angle of the plane upon which the object is supposed to be resting.

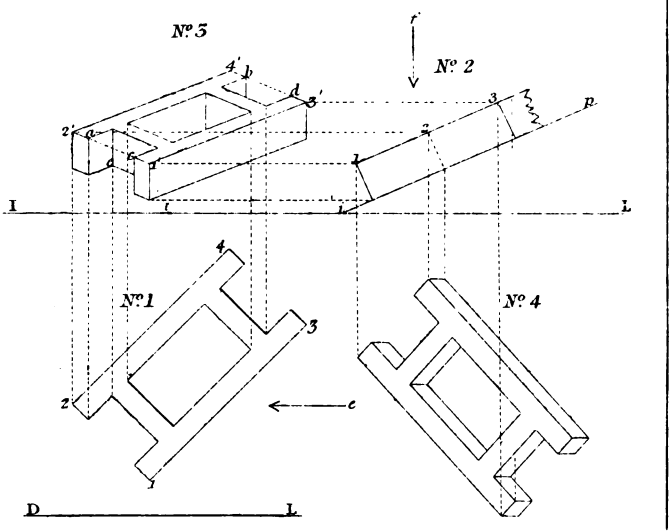
From what has been said of this species of projection, it is evident that the most natural way of proceeding with this example is to find the elevation or position of all the points upon the inclined plane by measuring their distances from the datum line. Many of the points, however, may be dispensed with, and a saving of labour effected.

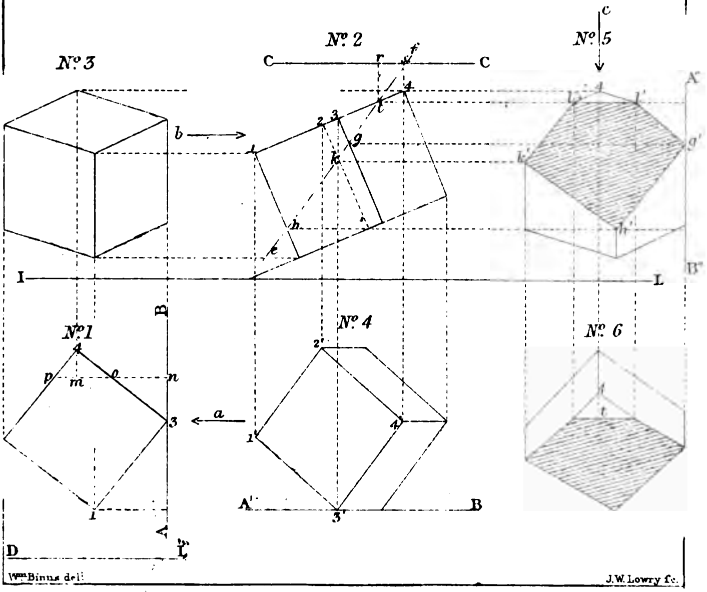
Find the position of points 1, 2, 3, upon the inclined plane No. 2. Through each point, and at right angles to the plane, draw a line equal in length to the breadth of wood of which the frame is made; and draw the line 1 3, parallel to *i p:* then will 1 2 3 represent the upper face of the figure. Find the projection of points 1, 2, 3, in the upper face of the figure, in points 1′, 2′, 3′, No. 3; and join them by right lines. From point 3′, No. 3, draw a line parallel to 1′ 2′, and from point 2′ a line parallel to 1′ 3′, the intersection of which with line drawn from 3′ will give point 4′. From point 1′ let fall a vertical line, and determine its length by drawing a line from lower end of line 1, No. 2, as shown by the projecting ray. From the lower end of line 1′, No. 3, draw lines parallel to 1′ 2′ and 1′ 3′; and complete the projection of a solid block, as at Fig. 43, by letting fall vertical lines from 2′, 3′. If imaginary lines be now drawn from 1 to 2 and 3 to 4 in the plan, we shall have points in those lines representing the thickness of each side piece; find the projection of those points or lines in No. 3 by drawing vertical lines from the plan cutting line 1′ 2′ in points *a*, *c*; and from *a* and *c* draw lines parallel to 2′ 4′, or 1′ 3′, cutting line 3′ 4′ in *b*, *d*: then will 2′ 4′, *a b*, and 1′ 3′, *c d*, represent the two side pieces,—the lines between which, from *a* to *c*, and *b* to *d*, are to be erased. Again, the position of the cross-bars will be found in No. 3 by erecting vertical lines from their corresponding points in the plan, as shown by projecting rays; and since the cross-bars are parallel to the imaginary lines 1 2, 3 4, in the plan, they will be parallel to corresponding lines in the projection. If a line be drawn from the lower extremity of line *a*, parallel to *a b*, meeting the vertical line of the first cross-bar in *e*, and a line from *e* parallel to the upper edge of the cross-bar, the projection of the figure will be completed from four points in the elevation on the inclined plane.

ART. 90.—From this problem we may deduce the following important theorem:—*All lines and planes which are parallel to each other in the original figure will be parallel to each other in the projection of that figure, whether such projection be perpendicular, horizontal, or inclined.*

No 4 represents the projection of No. 2 in the *lower*

PLATE 3





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J.W. Lowry fc.

plane, or its appearance when viewed at right angles to the intersecting line, as indicated by the arrow *f*. The attempt to obtain the projection of this figure may, however, be deferred until the student has read the explanation of the following problem, when he may return to the above as an example for practice.

PROBLEM XVIII

The plan of a cube and inclination of the plane being given to find the projection of the cube in the upper and lower planes, together with a sectional elevation and sectional plan of the cube.

ART. 91.—Let No. 1, on the lower part of Plate III., be the plan of the cube. The boundary lines of a cube being equal to each other (ART. 22), the lines 1, 2, 3, 4, No. 2, will be equal in length to 1 2, or 2 4, No. 1. Having drawn the elevation upon the inclined plane, and completed the *projection* No. 3 as directed (Problem XV.) for a rectangular block, we shall now proceed to get the *plan* or *projection* of No. 2 in the lower plane.

If point 1, 2, 4, or 3, in No. 1, represent a line, such line must be at right angles to the plane of projection; therefore every part of that line, when seen in the direction of the arrow *a*, will be the same distance from the eye. Moreover, since No. 2 is a view of No. 1 as seen in the direction of arrow *a*, line 3, No. 2, will manifestly be parallel to the vertical plane; and the plan of that line will consequently be a right line drawn parallel to the intersecting line (ART. 29).

ART. 92.—If the reader can now imagine a plane passing through line 3 parallel to the vertical plane, the projection of such imaginary plane in the lower plane of projection would be a right line, A′ B′, No. 4, drawn parallel to the intersecting line; and the position of that plane in No. 1 would be represented by a line, A B, drawn through point 3 at right angles to the intersecting line; because No. 2 is a view of No. 1 as seen in the direction of the arrow *a*, the imaginary plane being interposed perpendicularly between the eye and the object. From point 3, No. 2, let fall a vertical line, 3 3′, cutting A′ B′ in point 3′: then will 3′ be the plan of point 3. Now the plan of point 1 will be found by measuring the distance of that point from A B along the dotted line in No. 1, and setting that distance off from A′ B′ on the line 1 1′; because point 1 in No. 2 is farther from the eye than 3 by that distance: therefore 1′ is a plan of point 1. If points 2 and 4 be obtained in the same way, and the four points be joined by right lines, we shall have a plan of the upper face of the cube.

Again, since lines 2 and 4, No. 2, are parallel to line 3, every point in which is the same distance from the eye, they must be parallel to the vertical plane; therefore from points 2′ and 4′, No. 4, draw lines parallel to the intersecting line, and determine their lengths by letting fall vertical lines, as indicated by the dotted line drawn from the lower end of line 4, No. 2; join those points by lines which will be parallel to 3′ 4′, 2′ 4′; and the *plan* of the cube will be complete.

ART. 93.—It is now required to find a *sectional elevation* of No. 2 taken through the line *e f;* supposing that portion of the figure on the left hand of the line of section to be removed, and the remaining portion to be viewed as indicated by the arrow *b*, which is parallel to the plane of projection.

The reader is requested to turn to ART. 20, and peruse the explanation therein given regarding the elevation of those points and lines which are nearest to the eye when the object is viewed at right angles to the plane of projection, and also concerning the projection of those points when the object is viewed in the direction indicated by the arrow.

It will be evident that the imaginary plane before referred to will, in the sectional elevation, be on the right hand of the object to be delineated. Let A″ B″ represent the imaginary plane. Now the elevation of every point in No. 2 must necessarily be in a right line drawn from such point parallel to the intersecting line (Theorem 1, Chap. VI.): therefore the elevation of point *g* in line 3 will be *g*′ in the section No. 5. Again, what is the distance from the imaginary plane to point *h*? Bear in mind that No. 2 is an elevation of No. 1 as seen in the direction of the arrow *a;* and since *h* is a point in line 1, its distance from the imaginary plane will be equal to the distance from A B (No. 1) to point 1, measured on the dotted line at right angles to A B. From *h*, No. 2, draw *h h*′ parallel to the intersecting line, and set off from A″ B″ the distance taken from No. 1: then will *h*′ be the elevation of point *h;* and *g*′, *h*′, joined by a right line, will give the elevation of the line of section *g h*. The elevation of every point will be found in like manner;—that is to say, *from the point to be projected draw a horizontal line; ascertain the distance of such point from the imaginary plane by consulting No.* 1, *and set off that distance from* A″ B″ *on the projecting ray: then the point where it cuts the ray will be the projection of the original point.*

Although the foregoing sentence may be said to explain the projection of this figure, a description of the remaining portion is desirable. The line of section from *h* (No. 2) proceeds along the opposite face of the cube until it cuts the dotted line 2 in point *k*. The elevation of that point will therefore be found by measuring the distance from A B to point 2, No. 1, and setting off that distance from A″ B″ on a right line drawn from *k:* then will *k*′ be the projection of *k;* and *k*′, *h*′, joined, will be the projection of that portion of the section from *h* to *k*.

We now come to the point or rather points *l;* for we shall find that the section at this point will produce a right line parallel to the intersecting line. Measure the distance from 4 to l on the line 4 1, No. 2, and set off that distance from 4 to m in the original figure, No. 1; through m draw $m\ n$ at right angles to A B, cutting A B in n: then will $n\ m\ p$, No. 1, represent a line drawn from the imaginary plane, No. 2, at right angles to the vertical plane. Now, what is the distance from the imaginary plane, No. 2, to the first point in the line l? Manifestly $n\ o$, No. 1; therefore upon the projecting ray drawn from l set off from $A''\ B''$ the distance $n\ o$ in point l'. Again, the length of the line of section at l, No. 2, is equal to $o\ p$, No. 1; set off that distance from l' to l'', No. 5: then $l'\ l''$ will be the projection of point l, No. 2, and the point beyond it. Join $l'\ g'$, $l''\ k'$; and complete the section by drawing in the section lines as directed (ART. 12). Join l' 4, l'' 4; and l' 4 l'' will be the elevation of the triangular piece on the upper face of the cube from l to 4, No. 2, or o 4 p, No. 1.

Again, the lines 1, 2, 3, No. 2, corresponding to points 1, 2, 3, No. 1, are parallel to the plane of projection; therefore their projection in No. 5 will be in lines drawn from the points g', h', k', at right angles to the intersecting line (ART. 28); and their lengths will be determined by drawing lines from the lower ends of lines 1, 2, 3, No. 2, in the same manner as described for the projection of No. 3, ART. 83.

ART. 94.—There is one point to which we must direct the careful attention of the student. Line 1 4, No. 2, represents the upper face of the cube; that is to say, it represents a plane which is at right angles to the vertical plane of projection. The line of section $e\ f$ also represents a plane at right angles to the vertical plane. Now, when two planes which are at right angles to the vertical plane intersect each other, the line of intersection will be represented by a point, as l, No. 2; and the projection of that point in the upper or vertical plane will be a line drawn parallel to such plane and also to the intersecting line; and the projection of that line in the lower plane will be a line parallel to that plane and also to the intersecting line.

Art. 95.—We have now to find a *plan* of No. 5, or the appearance that figure would have if viewed in the direction of the arrow c, which is supposed to be parallel to the vertical plane.

From Theorem 2 it follows that the plan of point 4 must be somewhere in the projecting ray from that point. Let 4, No. 6, be the position of that point in the lower plane. Now the plan of points l'', l', will also be in lines drawn from those points at right angles to the intersecting line; the question, therefore, is (and these remarks apply to all the points in this figure), how are we to determine their position with regard to point 4? Remember that No. 5 is a view of No. 2, looking at that figure in the direction of the arrow b. It is evident, therefore, that point l is nearer to the eye than 4; but how much nearer? At any convenient distance above No. 2 draw a line, C C, *parallel to the direction of the visual rays;* and at right angles to that line draw lines from points 4 and l, cutting C C in $r\ s$: then will $r\ s$ represent the distance from point l to point 4;—that is, point l is nearer to the eye than point 4 by the distance $r\ s$. Therefore, since line $l''\ l'$, No. 5, is nearer the eye than point 4, it will, in the plan, be farther from the intersecting line (Theorem 5, Chap. VI.). Set off on the projecting ray from point 4, No. 6, the distance $r\ s$ in t; through t draw a line parallel to the intersecting line; and where it cuts the vertical lines from points l', l'', will be the projection or plan of points l', l''. Join the three points together; and the result will be a plan of the triangular corner on the upper face of the cube.

Art. 96.—The following rules for finding the plan of *any* given point, as in No. 5, may be of some service to the student when obtaining a similar view of the next figure:—

1st. Let fall a vertical line from the point of which a plan is required.

2nd. Find its corresponding point in the elevation on the inclined plane, as No. 2.

3rd. Ascertain the horizontal distance of the corresponding point from some given point in that elevation, as point 4.

4th. Having determined the position of point 4, in the lower plane, as in No. 6, set off therefrom the distance of the given point from 4 in the elevation (No. 2); draw a horizontal line through the point so found; and the plan of point sought will be where the vertical and horizontal lines intersect each other.

NOTE.—If vertical lines be drawn from the several points in No. 2 to meet the intersecting line, the distances of those points from point 4 can be measured upon that line, so as to dispense with the line C C. Moreover, the *sectional elevation* and *sectional plan* could have been much more readily described and worked out by a reference to Nos. 3 and 4; but such a course would interfere with the proper illustration of the principles of projection on the inclined plane; and it is much better practice for the student to find the projections of Nos. 5 and 6 from the *plan* and *elevation* only.

PROBLEM XIX.

Given the inclination of the plane and the plan of a cube with a block on each face,—the diagonal of the cube being at right angles to the intersecting line; required the projection of the cube and blocks in the upper and lower planes, together with a sectional elevation and sectional plan thereof.

ART. 97.—The following are the dimensions recommended for this figure:—

Each face of cube, 3¼ inches square.
Face of square block, 1⅝ inch square.
Thickness of block, $\frac{7}{16}$ of an inch

Presuming the elevation to be placed on the inclined plane, as shown at No. 2, Drawing D, and the *projection* of No. 3 delineated, as already explained for a plain cube, we shall now proceed with a description of the mode of drawing one of the blocks which project from the face of the cube.

If No. 2 were turned one-fourth of a revolution upon an axis parallel to the vertical plane, in the direction indicated by the arrow (ART. 74), so that every point in the figure would describe a plane at right angles to the original plane, and parallel to the intersecting line, the appearance of No. 2 would be precisely that of No. 3,—line 1 being in the centre of the cube, the face bounded by lines 1 and 3, No. 2, on the right hand, and the face beyond it being on the left hand of line 1, No. 3.* Therefore the projection of the blocks on those two faces of the cube which are presented to and nearest the eye in No. 3, will be obtained from that block marked E′, No. 2. We have been induced to mention this change of motion in the figure, from the fact that many students have made great efforts to get the projection of block E′ on the right-hand face, and of block F′ on the left-hand face of No. 3. Another reason for directing attention to this circumstance is, the importance of clearly understanding *the motion supposed to be given to these figures.* If No. 2, *with the plane upon which it stands,* were turned as just described, and then moved a certain distance to the left, every point and line in that figure would coincide with No. 3: consequently, block E′, No. 2, would be in the same vertical plane with block E, No. 1 (No. 2 being an elevation of No. 1, as seen in the direction of the arrow *a*). Moreover, if the dotted lines representing the projecting rays in No. 1 were produced to block G, every point in G would coincide with E: therefore every line in that block which is beyond E′, No. 2, is likewise coincident with E′.

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* It may be well to remind the student that points 1, 2, 3, 4, No. 1, represent lines, which are indicated in No. 2 by like figures, one figure only being used to represent a line (ART. 85)

It is also manifest that the projection of the block on the left-hand face of the cube can be obtained from E′ No. 2, and G, No. 1; that is, by drawing horizontal lines from E′, and vertical lines from corresponding points in G, as clearly shown by the projecting rays, and set forth in Theorem 3.

ART. 98.—When the object to be delineated is equilateral and equiangular, as a cube, the *projection* of No. 4 from Nos. 1 and 3 may be obtained by a different mode to that explained under Problem VIII., whereby the operation is somewhat expedited; but if the object to be projected is an irregular figure, the instructions given in Problem VIII. must be followed. It will be remembered that all those dimensions in No. 4 which are at right angles to the intersecting line were taken from point 3, No. 1, in the direction of the arrow *a*; but since, in the present figure, the distance from 3, No. 2, to the point beyond, is equal to 3 2, No. 1, and 1 4, No. 1, is also equal to 3 2, it will be evident that the plan of point 3, No. 2, and the point beyond can be obtained by letting fall a vertical line from 3, and drawing horizontal lines from 1 and 4, No. 1,—their intersection being the plan of point 3, and the point beyond it. The plan of points 1 and 4, No. 2, will be obtained in a similar way, by drawing vertical lines from those points, and horizontal lines from point 2 or 3, No. 1. Again, the length of the vertical boundary lines 3 and 4, No. 2, will be obtained by letting fall vertical lines from the lower end of such lines 3 and 4, which will complete the projection of the plain cube, that is, point or line 3, No. 4, will be a plan of 3, No. 2, point 4, a plan of 4, No. 2; and so on. In other words, if we suppose, No. 2 to be a perfect solid and attached to the vertical plane, and that plane to be bent in some line parallel to the intersecting line, so as to stand at right angles to the horizontal plane (No. 2 being as it were suspended above No. 4, and viewed at right angles to the lower plane), every point and line in No. 2 will coincide with No. 4. The figures of reference, 1, 2, 3, 4, would also coincide with No. 2, but not with No. 1. In order that they may do so, however, the student is requested to transpose the figures of No. 1 in his drawing, so that they may agree with No. 4: that is, 3, No. 1, will be substituted for 4; 2 for 1; 4 for 2; and 1 for 3. If the reader will make the same alteration with his pencil upon the drawing before him, the figures so transposed will agree with the following observations.

ART. 99.—It is a remarkable fact that although many students can readily obtain a plan of the faces of the cube from lines 1 3, No. 2, and 3 4, No. 1, they cannot produce *a plan of the faces of the blocks*, notwithstanding the two operations are identical. This arises from the difficulty of determining which of the points and lines in the elevation correspond to those in the plan; and this, after all, is really the only difficulty with all figures; for the projection of any two given points would, according to Theorem 3, appear a very simple matter: let us see, therefore, if we cannot make the other matter equally so. According to the relative positions of No. 2 and No. 4, the projection of the block on that face of the cube whereof 3 4 is the upper boundary line, will be obtained from F′, No. 2, and E, No. 1; and the projection of the block on that face of which 2 4, No. 4, is the upper boundary line, will be obtained from F′, No. 2, and F, No. 1. Taking the former for our illustration, and *t*, No. 2, as the first point of which a plan is required, the question is, what point is *t*, and how are we to find its corresponding point in the plan? To make this perfectly clear, it will be convenient to suppose the block to have two faces, an outer and inner face, the latter joining the face of the cube. This being understood, the answer is, that *t* is *one of the upper outer corners of the block;* and it is, moreover, that corner which is nearest to point 4. Again, *v* is an outer corner of the block nearest to point S, and *u* an inner corner nearest to point S. [See projecting rays from *t*, *v*, *u*, No. 1.]

It now remains for the student to find the projection of the two blocks F′, No. 2, and F, No. 1, according to Theor. 3, and then to ascertain whether any portion of block L′, No. 2, and the one beyond E′ (corresponding to H in the plan), will be seen; or, in other words, whether in No. 4 any portion of block E′ will project beyond the boundary line 1 S. From *x*′, No. 2, which represents the outer corner nearest to point 1, let fall a vertical line; then from *x*, No. 1, which represents the corresponding point, draw a horizontal line, and the point of intersection will determine whether any and what portion of the block can be seen. The completion of the remaining portion of the figure is left for the student.

NOTE.—As a test of careful and accurate drawing, the student will not meet with a better example than the projection of No. 4; for if all the points be correctly found *by projection*, and those points joined by right lines, every parallel line in the original figure *will be parallel in the projection of that figure* (ART. 90). To accomplish this, however, the manipulation must be very delicate, each line should be as fine as possible, and the projection of each point must be taken, not from one side, although they may touch, but from the centre of each line; otherwise the projected lines in No. 4 will not be parallel. The original figure should also be a perfect square.

ART. 100.—We now come to the *sectional elevation* and *sectional plan* of No. 2, taken through the line of section *e f*.

The student should close this book, and make the best attempt in his power to produce the elevation and plan; after which he may peruse the following explanation,—reference being made to Nos. 2 and 3.

The line of section first comes in contact with the vertical corner of the block on the dotted line 4, No. 3, and traverses the back edge of that block until it reaches the upper face of the cube; from that point it runs along the upper face of the cube parallel to the intersecting line, and cuts the boundary line 3 4 in point 6; its next course is along the back face of the cube until it cuts the vertical boundary line 3 in point 7; whence it passes along the front face of the cube until it comes in contact with the block E. Now the upper edge of the block being parallel to the upper face of the cube, the projected line of section is parallel to the line drawn on the upper face of cube to point 6 (ART. 94). The line of section then proceeds from the upper right-hand corner of the block, along its front face, to point 8, and thence along its front edge to 9, terminating at point 10 in line 1. The line of section takes the same course on the opposite or left-hand side of the cube. The elevation and plan of each point and line are found as already described for a plain cube.

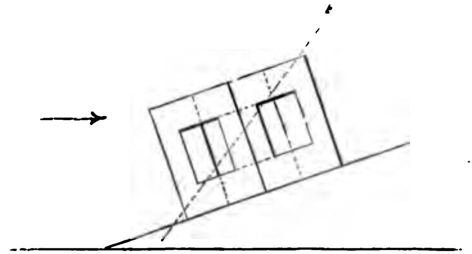
Those persons who are desirous of further practice with this figure are recommended to take in hand the hollow cube with blocks, and endeavour to make a drawing of the *six* views (four only of which, including sectional elevation and sectional plan, will be found on Drawing E); *g h* being the line of section. We may here remark that in general about 20 per cent. of the students who have gone through this course of instruction, including the last figure, have made correct drawings of the hollow cube without any assistance whatever from the writer. If the reader should be of opinion that more space than necessary has been devoted to the consideration of cubes and blocks, he is earnestly recommended to try the next Problem; and on referring to Nos. 1 and 2, Drawing E, to test the accuracy of his work, the result will convince him that no unnecessary examples or explanations have been given.

PROBLEM XX.

Required the sectional elevation and sectional plan of a skeleton cube.

ART 101.—Fig. 45 represents a skeleton cube, of which it is required to obtain a sectional elevation, as seen in the direction of the arrow, and also a plan of the sectional elevation.

Fig. 45.



The cube for this example is recommended to be made $3\frac{1}{4}$ inches square, with an opening through the centre of each face $1\frac{5}{8}$ inch square. If the inclination of the plane be 25°, and the line of section A B be drawn through the points shown at Fig. 45, the accuracy of the work may be tested by referring to Drawing E.

PROBLEM XXI.

The plan and elevation of a flight of steps being given, to find the projection thereof upon the upper and lower inclined planes, the projections to be worked out from the points and lines given in the plan and elevation.

ART. 102.—Let No. 1, Drawing F, represent the plan of a flight of steps, with a wall on the right, as described in ART. 58; and let No. 2 be an elevation, as seen in the direction of arrow *a*. Presuming the rise of each step to be equal to two-thirds of its horizontal face, and the upper face of the wall to be level with the top step, the length of the lines represented by points 1, 2, 3, No. 1, will be equal to two-thirds of the right line 3 a. Find the elevation of lines 1, 2, 3, No. 2, by ART. 81; and draw the right line 1 3, to represent the upper face of the wall and top step. It is now required to find the projections No. 3 and No. 4 from nine given points; namely, 1, 2, 3, a, f, in plan, and 1, 2, 3, g, in elevation.

ART. 103.—For No. 3. Find, by Theor. 3, the projection of points 1, 2, 3, in points $1'$, $2'$, $3'$. Find the projection of g, the lower end of line 1, in point g', No. 3. Join $2'$ $3'$; and from each of the points $1'$ g' draw a line parallel to $2'$ $3'$. Join $1'$ $2'$; and parallel thereto draw lines e $3'$, g' g''. Let fall vertical lines from $1'$ $2'$; and from points f and a, No. 1, draw lines at right angles to I L, cutting lines drawn from $1'$ and $3'$, No. 3, in points e and f'. Make the vertical line drawn from point a, No. 1, equal in length to vertical line $1'$ g', No. 3; and divide it into five equal parts, in points a, b, c, d, to represent the rise of each step. From each of the points a, b, c, d, draw lines parallel to $1'$ $2'$, or e $3'$. Divide $3'$ e also into five equal parts in b', c', d', e'; and let fall perpendicular lines, which will represent the vertical corners of the steps. If a line be now drawn from each of these corners parallel to $2'$ $3'$, as e' e'', the projection of No. 3 will be complete.

NOTE.—Instead of dividing the line e $3'$ in points b', c', d', e', the projection of those points can be obtained from corresponding points in the plan.

ART. 104.—For No. 4. In finding the plan of No 2, as shown in No. 4, we must have recourse once more to an imaginary plane, supposed to pass through line 2, No. 2, as described in ART. 92. Through point 2, No. 1, draw A B at right angles to the intersecting line. Parallel to the intersecting line, and at any convenient distance therefrom, draw A′ B′, to represent a plan of the imaginary plane; then will the projection of the upper and lower ends of line 2, No. 2, be found by letting fall vertical lines from those points to cut A′ B′ in points $2'$, $2'$. From point 1, No. 2, let fall a vertical line; measure the distance of that point from A B, No. 1, and set it off from

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A′ B′, No. 4, in point 1′: then will 1′ 2′, joined, be the projection of line 1 2, No. 2. Find the plan of point 3 in like manner in 3′. From 3′ draw a line parallel to I L, and determine its length by letting fall a vertical line from the lower end of line 3, No. 2 (ART. 92); join 2′ 3′, 2″ 3″: then will 2′ 3′, 3″ 2″, be a plan of the vertical wall or end of the steps. From 3′ and 3″ draw lines parallel to 1′ 2′, of indefinite length; measure the distance from A B to point *a*, No. 1, and set that off on any line at right angles to A′ B′, as from A′ in point *h*. If a line be now drawn from *h* parallel to A′ B′, its intersection with the line drawn from point 3′ will give the plan of that point corresponding to *e*, No. 3. Produce *h e*, No. 4, to meet the line drawn from 3″, and divide the line so produced into five equal parts in points *a, b, c, d;* from each of these points draw lines parallel to *e* 3′, and they will represent the horizontal corners or ends of the steps. Divide *e* 3′ also into five equal parts, in points *b′, c′, d′, e′;* and from each point draw a line parallel to 3′ 3″ or A′ B′, to represent the vertical corners of the steps, as clearly shown. To find the position of point *f′*, measure its distance from A B, No. 1; set off such distance from A′ B′ No. 4, on the line A′ *h*, and draw a line therefrom parallel to the intersecting line; if a line be now drawn from 1′, No. 4, parallel to 2′ 3′, its intersection with the line just drawn will give the position of point *f′*. From *f′* draw a line of indefinite length parallel to 1′ 2′. If a line be now drawn from *e′*, No. 4, parallel to 2′ 3′, it will represent the front edge or corner of the top step, and will also determine the length of line drawn from point *f′* in *e″*. From each of those points representing the corners of the steps, draw lines parallel to 2′ 3′, making each line of the same length as *e′ e″;* and from the end of each line so formed draw a line, as *a″ b″*, parallel to *f′ e″*. The projection of the steps in the lower plane will then be complete.

ART. 105.—The projection of every point in No. 4 can be obtained by attention to the following rules:—

1. Measure the distance of the point sought from D L, and set off that distance upon the inclined plane.

2. From such point on the inclined plane draw a line at right angles thereto, making it equal in length to the vertical height of the original point from its plane of projection No. 1.

3. Having found the position of the point in No. 2, let fall a vertical line; and the position of the point sought will be found in that line, by measuring the distance of the original point from A B, No. 1, and setting off that distance from A′ B′, No. 4: the point of intersection will be a plan or projection of the given point.

It may be observed that No. 1 admits of being turned in a variety of ways, and that as many different figures will be produced: for instance, the wall in the original figure may be placed on the left hand, or the steps turned end for end, or placed at various angles with the ground line; all of which will be found excellent practice for the student.

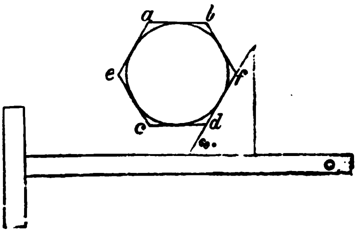
PROBLEM XXII.

To draw a regular hexagon.

ART. 106.—The practical mode of describing a hexagon or six-sided figure (such as the nuts of bolts for holding together parts of machinery) is as follows:—

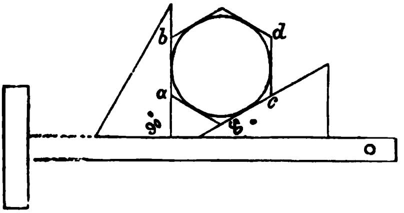
Describe a circle of any given diameter, and with the T square draw two tangential lines, *a b*, *c d*, Fig. 46. Apply the set square of 60° to the edge of the T square, and draw parallel tangential lines *e a*, *d f*, by sliding the set square along the edge of the T square. If the set square be now turned over and

Fig. 46



again applied to the edge of the T square, parallel tangential lines *e c*, *b f*, may be drawn, and the six-sided figure completed.

Fig. 47.



When two sides of the nut or hexagon are vertical, as shown at Fig. 47, the angle of 30° is employed; the lines *a b*, *c d*, being drawn by applying the set square as shown on the left hand of the figure.

Problem XXIII.

Given the plan and length of an hexagonal prism, to find the projection thereof under the following conditions.

Art. 107.—Required: 1st. An elevation in the vertical plane.

2nd. The same elevation on a plane which makes an angle of 30° with the intersecting line.

3rd. The projection in the lower plane of the prism and the plane on which it stands.

4th. The projection of the last-mentioned figure in the upper plane; supposing the prism with its plane to have been turned one-eighth of a revolution upon an axis perpendicular to the horizontal and parallel to the vertical plane.

Having drawn a plan of the prism, as No. 1, Drawing G, the elevation in the vertical plane, as No. 2, will be obtained by drawing lines from points 1, 2, 3, 4, and making them equal in length to the height of the prism. The upper surface of the prism will be represented by drawing line 1 4 parallel to the ground line. It must be understood that No. 2 is not a projection upon an inclined plane, but a projection in the vertical plane: therefore the lines 1, 2, 3, 4, representing the angles of the prism, are parallel to the plane of projection.

No. 3 represents the same elevation as No. 2, supposing that figure to rest upon a plane which makes an angle of 30° with the intersecting line: in other words, No. 3 is a copy of No. 2, every line in which may be drawn by applying the set square of 30° and 60° to the edge of the T square. It is now required to find a plan of No. 3, or its projection in the lower plane.

Since No. 3 is the same elevation as No. 2, or a view of No. 1 as seen in the direction indicated by the arrow, the points 2 and 3 are nearest the eye; and the distance from those points to the points beyond them, which we shall call 2′ 3′, is equal to 2 2′, 3 3′, No. 1: therefore the plan of No. 3 will be obtained by drawing vertical lines from the several points in that figure, and horizontal lines from corresponding points in No. 1, as clearly shown by the projecting rays. From A and B let fall vertical lines, and make A A′, B B′, No. 4, equal to the width of the plane on which No. 3 is supposed to stand; draw A′ B′, A B: then will No. 4 be a plan of a prism resting upon a plane which makes an angle of 30° with the horizontal plane: therefore the least angle which the axis of the prism makes with the horizontal plane will be 60°, and the greatest angle 120°, as shown by the dotted line in No. 3.

Art. 108.—It is now required to find the projection of No. 3 in the upper plane, or the appearance that figure would present if it were turned in the direction of the arrow, together with the plane on which it stands, so as to make one-eighth of a revolution.

The direction of motion in this case is left-handed (Art. 74). Again, the arrow is drawn parallel to the intersecting line and at right angles to the vertical projecting rays, to indicate that the figure is supposed to turn upon an axis, such as that represented by the projecting ray from point 2; in which case every point in the figure would move in a plane at *right angles* to the vertical plane and *parallel* to the horizontal plane. Again, the amount of motion is one-eighth of a revolution, that is $\frac{360}{8} = 45°$. Therefore, at an angle of 45° with the intersecting line draw a line, 1 4, No. 5, of indefinite length; and let that line represent a centre line which would correspond to a line drawn through the same points in No. 4. Make No. 5 an exact copy of No. 4. Then, since we have in the lower plane a number of points, 1, 2, 2′, 3, 3′, 4, and points corresponding to them in the upper plane, the projection of these points will be found by Theorem 3; and if the points so found be joined by right lines, the projection of the upper face (No. 6) will be complete.

The student must not forget that points 2 and 3, No. 3, represent those points in the plan marked 2′ and 3′, as well as points 2 and 3.

Having found the projection of the upper face of the figure, as shown at No. 6, the projection of the lower face will be found in like manner. Join the upper and lower faces by right lines. Then will No. 6 be a correct projection of No. 3, or the appearance that figure would present if it were turned through an arc of 45° upon an axis parallel to the vertical plane and at right angles to the ground line.

PROBLEM XXIV

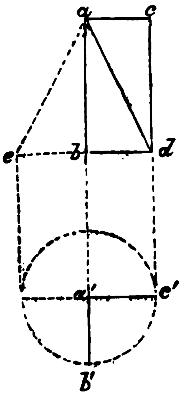
Required the angle which the axis of the prism No. 6, in the last problem, makes with the vertical and horizontal planes.

ART. 109.—Before we enter on a demonstration of this problem, it will be necessary to direct attention to what is called, in geometrical language, the "generatrix" of a solid.

termination

Let *a b d c*, Fig. 48, represent a plane, of which *a′ c′* is a plan. If the point *c′* be caused to rotate upon *a′* as a centre, *c′* will describe a circle, every point in which will be the same distance from the centre *a′*. Again, if the line *c d* be carried round *a b*, as an axis of rotation, the figure described or generated by *c d* will be a cylinder, provided *c d* be parallel to *a b*: wherefore *c d* is called the "generatrix" of a cylinder, and *a c*, *b d*, the "directrix." Remove *a c*, *c d*, and draw the right line *a d*. Then will *a′ c′* be a plan of *a d*; but the solid generated by *a d* will be a cone, of which *d e* will be the base: consequently, *a d* and *a′ c′* may each be considered as the generatrix of a cone, inasmuch as *a′ c′* is a plan of *a d*. Moreover, since the point *c′* is the same distance from *a′* during every part of a revolution, and the surface of the cone *e a d* is generated by the line *a d*, it follows that every right line drawn from the base to the apex of a cone must be of the same length: that is to say, the actual length of any right line, as *a b*, drawn upon the surface of a cone, will be equal to *a d* or *a e*; because *a′ b′*, which is equal to *a′ c′*, is a plan of *a b*. Therefore, every right line which can be drawn from *a* to meet the base *e d*, will be of the same length; and the angle which that line makes with the base of the cone will always be equal to *b d a*.

Fig. 48.



After the above explanation, the student will be able to see his way more clearly through the problem relating to the hexagonal prism.

ART. 110.—Let *a*, No. 1, Plate 4, represent the axis of the prism; *a b*, No. 2, its elevation in the vertical plane; and *a b*, No. 3, the same elevation as No. 2, but inclined to the intersecting line at an angle of 60°, as shown by the dotted line from *a b* produced.

Find the projection of the two points *a*, *b*, in Nos. 4, 5, and 6, as directed for Prob. XXIII.; and join those points by right lines. Then will *a′ b′*, No. 4, represent a plan of *a b*, No. 3, and *a b″*, No. 5, the position of *a′ b′*, No. 4, after moving on *b′* as a centre through an arc of 45°, as shown by the dotted line *a″*, *b′*. Again, *a b*, No. 6, being obtained from corresponding points in Nos. 3 and 5, the projection of No. 6 will be the position of *a b*, No. 3, after making one-eighth of a revolution (ART. 108).

Through *b*, No. 3, draw a line, *c b*, at right angles to the intersecting line: *c b* may, therefore, be said to represent an edge view of the vertical plane. Then, since *a b* makes an angle of 60° with the ground line, the angle *c b a* will be $90° - 60° = 30°$. Again, *a′ b′*, No. 4, is a plan of *a b*, No. 3: therefore, *a′ b′* makes an angle of 60° with the horizontal and 30° with the vertical plane, represented by the dotted line *b′ s′*. But in obtaining the projection of No. 5, *a′ b′* is supposed to have been moved into the position *a″ b′*; and *a b*, No. 6, is the projection of *a b″*, No. 5, in that position. We will now suppose *a b*, No. 3, to be the generatrix of a cone, *a b p*, the base, *a p*, of which is parallel to the plane *c b*. It is evident, from what has been said, that the angle formed by *a b* would be the same during every part of its revolution.

Again, let *a b*, No. 6, be moved in like manner on *b* as a centre, so that the point *a* would describe a circle, *a f g*, parallel to the plane of projection. Through *b″*, No. 5, draw *h b″ i* parallel to the intersecting line: then will *h b″ i* represent a plan of the vertical plane in which the point *b″* revolves. From *f* and *g*, the extremities of the circle described by point *a*, let fall vertical lines, *f f′*, *g g′*; and through *a*, No. 5, draw *f′ g′* parallel to *h i*: then will *f′ a g′* be a plan of the circle described by point *a*, No. 6, because the distance of that point from the vertical plane is equal to *a k*, No. 5. Let *a b*, No. 6, be moved into the position *f b*: then, since *f′* is a plan of *f*, and *b″* a plan of *b*, if *f′ b″* be joined by a right line, *f′ b″*, No. 5, will be a plan of *a b*, when in the position of *f b*, No. 6; and since *h f'* is equal to *a k*, the distance of point *a* from the vertical plane *f' b'' h* will be the angle which *a b*, No. 6, makes with that plane.

ART. 111.—It can likewise be proved that *a b''*, No. 5, makes an angle of 60° with the horizontal plane.

Referring to No. 4, it will be seen that *a'* has been turned on *b'* as a centre, in the direction of *a' a''*: if that motion were continued, *a'* would describe a circle parallel to the horizontal plane. Through *a*, No. 3, draw *a m* parallel to the intersecting line; and make *c m* equal to *a c*. Then will *a m* represent the base of the cone described by the generatrix *a b* moving on *c b* as an axis; and, since every right line drawn from the base to the apex of a cone makes the same angle with the base, it follows that *a' b'*, No. 4, will make the same angle with the horizontal plane at every part of its revolution. Therefore, *a b*, No. 6, makes an angle with the vertical plane equal to *h b'' f'*, No. 5, and an angle of 60° with the horizontal plane.

ART. 112.—The angle which *a b*, No. 6, makes with the vertical plane may also be obtained from a side elevation of the line *a b* as well as from the plan. If *a b*, No. 6, were moved into the position *r b*, the point *r* would still be the same distance from the plane of projection as *a*. From *r* draw *r s t* parallel to the intersecting line; make *s t* equal to *a k*, No. 5, or *a'' s'*, No. 4; and join *t b*. Then will *t b*, No. 3, be a side elevation of *a b*, when in the position *r b*, as seen in the direction of the arrow; and since *s b* represents an edge view of the vertical plane, and *t b* the same elevation of *r b*, *t b s* will be the angle which *a b*, No. 6, makes with that plane. Although much circumlocution might have been avoided in describing the angle which *a b*, No. 6, makes with the vertical plane, it is presumed that the mode of reasoning adopted will be found of service where problems of this nature are required to be solved. As an illustration of the simplest way of arriving at a solution of this problem, we will once more direct attention to *a b*, No. 6, as the generatrix of the cone *x b y*. If the line *x y* be drawn at *right angles* to *a b*, and *a x*, *a y* be each made equal to *a k* (the distance of *a* from the vertical plane), and *a b* be supposed to turn on *b* as a centre in such manner that *a* will describe a plane, *a x y*, at right angles to the plane of projection, then will *x b a* or *y b a* represent the angle which *a b* makes with the vertical plane, which is about 20° 30′.

A little attention to the foregoing problem will enable the student to determine the actual length of any line or lines in Nos. 5 and 6, Drawing G, and the angle which such line or lines may make with the planes of projection. We can therefore employ this problem for determining the angle which the rays of light, mentioned in ART. 60, make with the vertical and horizontal planes of projection.

CHAPTER VII.

ON THE PROJECTION OF CURVED LINES.

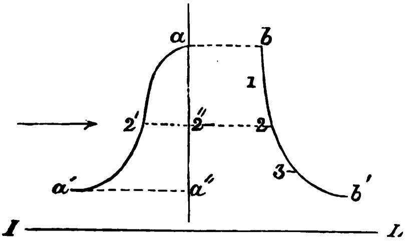
THE principles which govern the projection of curved lines differ in no respect from those which regulate the projection of right lines, inasmuch as a curved line is nothing more than a line composed of an infinite number of points. It would therefore appear, that if the projection of a point on the vertical, horizontal, and inclined planes be thoroughly understood, the projection of a curved line would be a simple matter; but it is not so in all cases. The question naturally arises, where lie the difficulties with which the student may have to contend in the present Chapter? The answer is, not in the projection of a single point; but in obtaining a perfect knowledge of the figure required to be delineated, and the relation or position which one point in a line bears to another, or, in other words, *the distance from that point to the point beyond it.* The whole subject of orthographic projection may almost be said to be comprised in the above simple sentence; and every difficulty with which the student may meet will be removed if he can only answer that question when the projection of any point is required. In confirmation of these assertions, we have selected as our first example the projection of a compound curved line, taken from a portion of the framework of a machine, which the student will be better able to realise if he take a piece of lead or copper wire and bend it in such manner as to produce the form exhibited in the next wood-cut.

PROBLEM XXV.

Given two elevations of a curved line taken exactly at right angles to each other, to find the projection thereof in the lower plane.

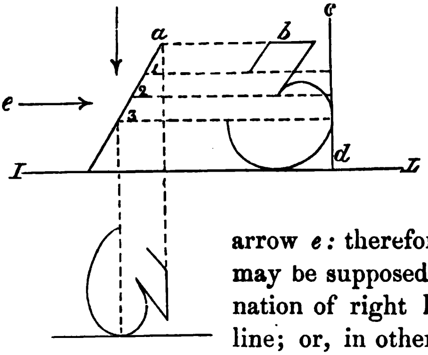
ART. 113.—Let *a a′*, Fig. 49, be a front elevation of the curved line, and *b b′* an edge view of *a a′*; that is to say, *b b′* is a view of *a a′*, or the appearance that figure would have if seen in the direction of the arrow: therefore, the lower end of line *b′* is nearer to the eye than the upper end, because *a′*, the corresponding point, is nearer the point of sight than *a*. If any number of points, as 1, 2, 3, be taken in *b b′*, there will be a certain distance from each of those points to some *fixed point, line, or plane beyond them;* and this is precisely what we have to determine. From *b* let fall an imaginary line parallel to the plane of projection: such a line would be correctly represented by letting fall a vertical line from *a*. Now what is the distance from point 2 to the point beyond, or to the imaginary line let fall from *b*? Manifestly 2′ 2″. For the same reason the distance from *b′* to the point beyond is equal to *a′ a″*; and so on with points 1 and 3. Find, therefore, the projection or plan of the several points, according to Theorem 5; and through the points so found draw the curved line, which will be a plan of *b b′*, or the appearance of the curved line as seen from above.

Fig. 49.



Art. 114.—Another illustration, of a more familiar kind, is the following:—

Fig. 50.



Let the right line *a*, Fig. 50, represent an edge view of the Figure *b;* that is to say, *b* is the appearance which *a* would present if seen in the direction of the arrow *e:* therefore the straight line *a* may be supposed to contain a combination of right lines with a curved line; or, in other words, the line *a* is supposed to contain the whole of the lines in *b*. It is now required to find the projection of *a* in the lower plane.

From what has been said concerning the relative positions of two figures or elevations in the vertical plane, it will be understood that the convex portion of the Figure *a* is that which is nearest to the eye: such portion, in plan, will therefore be farthest from the intersecting line, Theorem 5, Chap. VI. Take any number of points, as 1, 2, 3, in line *a,* and draw lines therefrom parallel to the intersecting line, cutting Figure *b* in corresponding points. If an imaginary plane, parallel to the vertical plane of projection (Art. 92), be supposed to pass through point 3, which is the most prominent in the figure, an edge view of such a plane will be correctly represented by *c d;* and the distance from that imaginary plane to any point in line *a* can be ascertained by measuring its distance from *c d,* and a correct plan obtained.

Art. 115.—It has been stated (Art. 32) that there are exceptions to the rule of making all elevations in the vertical plane; and, as we may now find it convenient to depart from that rule, we will give as an illustration the following example:—Suppose A to be a plan of a building or machine

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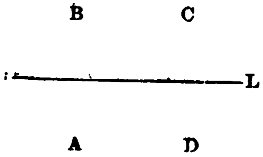
whereof a front elevation, end elevation, and transverse section are required. The front elevation would be represented at B; the end elevation at C; and the transverse section would be most conveniently drawn in the position D. But A is a plan; whereas D is a sectional *elevation* of A: therefore we have a plan and elevation in the lower plane Such an elevation may also, if required. be given on the left hand of A, as in the next problem.



PROBLEM XXVI.

Given the plan and elevation of a curved line in combination with a right line, to find the projection thereof, when making any given angle with the vertical and horizontal planes.

ART. 116.—Let *a b*, No. 1, Plate 5, represent a plan of the compound line, of which A *b′* C, No. 2, is an elevation, as seen in the direction indicated by the arrow *d*: that is to say, that portion of the original line from *a* to *b* is of the same curvature as A *b′*, No. 2; and from *b*, No. 1, to the point beyond it is a straight line, the length whereof is equal to *b′* C, No. 2. It is now required to find an elevation of the original line *a b* in the upper plane, as seen in the direction of the arrow *e*.

Take any number of points, 1, 2, 3, in *a b*, and draw lines therefrom parallel to I L, cutting A *b′*, No. 2, in corresponding points. Produce C *b′* indefinitely; and at right angles to the line so produced draw lines from those points in the curved line corresponding to 1, 2, 3, cutting C *b′* produced in points 1′, 2′, 3′, *a′*. In the vertical plane, and at right angles to I L, draw A′ C′, No. 3, equal in length to *a′* C, No. 2; and set off from C′, on the line A′ C′, No. 3.

termination

the points *b″*, 1, 2, 3, equal to *b′*, 1′, 2′, 3′, in No. 2. We have now got a certain number of original points in the lower plane, and points corresponding to them in the vertical plane, the projection of which will be found by Theorem 3. To remove any difficulty, however, we may observe that the points 1, 2, 3, in the original line (No. 1) occupy a certain position with regard to some other point, such as *b*: that is to say, point 3 is further from the plane of projection, or nearer to the eye, than point *b*, by the distance *b′* 3′, No. 2; and so on with points 2 and 1. Therefore, since 1′, 2′, 3′, No. 2, represent the vertical heights of those points, which have been set off upon A′ C′, No. 3, their projection in the upper plane, as shown at No. 4, will be found as directed for the projection of a point.

Draw *b c*, No. 5, at any angle to the intersecting line; and make *a b c*, No. 5, a fac-simile of No. 4. This will be best accomplished by producing *c b* indefinitely in both figures. Set off from *b*, No. 5, on *c b* produced, the same vertical heights 1′, 2′, 3′. Through such points draw ordinates or lines at right angles to *b c*; make each of the lines so drawn in No. 5 equal to their corresponding lines in No. 4, as 3′ 3, No. 4, and 3′ 3, No. 5; and through these points draw the curve *a* 3 2 1 *b*. The plan of No. 5 will now be found as directed for the projection of a point (Theorem 3), and clearly shown by projecting rays from Nos. 1 and 5 to No. 6, which is a plan of No. 5.

We will now suppose No. 5, as in the case of the hexagonal prism, to move upon *c* as a centre, in such manner that every point will describe a circular plane parallel to the intersecting line, and at right angles to the vertical plane,—the motion being right-handed; in which case the point *a*, No. 5, would recede from the eye and approach the vertical plane. The direction of such motion would be clearly indicated in No. 6 by the arrow (ART. 73). Let *a x*, No. 6, be the extent of the motion of No. 5. Required its projection in the upper plane under these circumstances.

Parallel to the intersecting line draw *a′ c′*, No. 7, indefinitely; from *c′*, with a radius equal to *c a′*, No. 6, describe the portion of a circle *a′ x′*; with the distance *a x*, No. 6, set off *a′ x′*, No. 7; and join *x′ c′*. Then will *x′ c′*, No. 7, be the position of *c b a′*, No. 6, after making *a x* portion of a revolution. Set off the distances *c*, *b*, 1′, 2′, 3′, *a″*, No. 6, upon *c′ x′*, No. 7; at right angles to *c′ x′* draw the ordinates; and proceed with a copy of No. 6 as directed for No. 5. We have now got in No. 7 a series of points, *c*, *b*, 1, 2, 3, *a*, corresponding to similar points, *c*, *b*, 1, 2, 3, *a*, in No. 5, the projection of which will be found in the upper plane by Theorem 3. Therefore *a b c*, No. 8, is the projection of No. 5 turned upon *c* as a centre, through *a x* portion of a revolution.

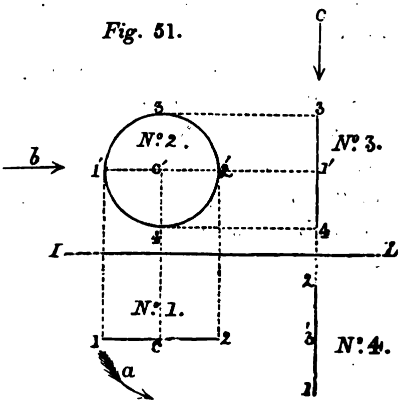
ART. 117.—If the student will now refer to Drawing H, he will there see an application of the foregoing principles, which will afford him a good subject for practice. As a first step, however, he may take the upper portion of the figure only, which is nothing more than a repetition of the preceding problem on six right lines, as 1, 2, 2′, &c., No. 1; and we would here caution him against the use of a separate letter or character to denote each point of which the projection on the several planes is required, as such a course would involve him in difficulties sufficient to disgust the most patient and persevering. The simplest way of going to work is to consider all the angles of the hexagonal block (of which No. 1 is the original figure or plan) as so many principal lines, numbered 1, 2, 2′, 3, 3′, 4, and having determined upon the points to be taken in each line, he has then got points 1, 2, 3, 4, 5, 6, 7, 8 in line 1, points 1, 2, 3, 4, 5, 6, 7, 8 in line 2′, and so on in lines 3′ and 4. By this course we get the whole number of points, amounting to about 72, and no less than 60 lines in each figure, visible and invisible, clearly and systematically denoted by eight characters or figures. Again, in finding the projection of No. 1 in the upper plane, it is only necessary to obtain by projection the several points 1, 2, 3, 4, 5, 6, 7, 8 on lines 1 and 2′. Having done this, draw the centre line A B. Then com mence with No. 3, by drawing the vertical heights parallel to the inclined plane; and at right angles to the plane draw the centre line A′ B′. Now proceed to set off on each side of the line A′ B′ the distance of the several points from the centre line A B, No. 2. Whilst the compasses are set for any particular point, the same distance may be marked off on the right hand of A B, No. 2. This mode of proceeding may also be practised with Nos. 4 and 5; but the projection of No. 6 must be obtained by Theorem 3, as already described. It will be observed that the projecting rays from points 3, 4, 5, 6, 7, 8, on line or angle 2′, are shown in No. 2, and that the projecting rays from the same points on line or angle 3, are exhibited in Nos. 3, 4, 5, and 6. Much time, likewise, may be saved in the projection of these figures by a peculiar mode of manipulating with the set squares, which can only be learned by practice or from oral instruction.

PROBLEM XXVII.

Given the edge view of a circular plate in the lower plane, to find its projections.

ART. 118.—If 1 2, No. 1, Fig. 51, represent the edge view of a circle, whereof 1 *c* or 2 *c* is the radius, the elevation of such a figure would be obtained by drawing a line from *c* at right angles to I L. From any given point in the line drawn from *c* (as *c′*), with a radius equal to 1 *c* or 2 *c*, describe the circle 1′ 3 2′ 4: then will 1′ 3 2′ 4

Fig. 51.



be the elevation of the right line 1 2, which is said to be the plan or edge view of a circle.

Since the circular plate in No. 2 is parallel to the plane of projection, an edge view, as seen in the direction of the arrow *b*, would also be a right line. From points 3 and 4 draw lines parallel to I L, and at right angles thereto draw 3 4, No. 3: then will No. 3 be an edge view or elevation of the circular plate in the vertical plane (ART. 28).

Again, a plan of No. 3, as seen in the direction of the arrow *c*, will also be a straight line, equal in length to the diameter of the circle; and since No. 3 is at right angles to the plane of projection, No. 4, which is a plan of No. 3, will be at right angles to the intersecting line, and also to its plane of projection.

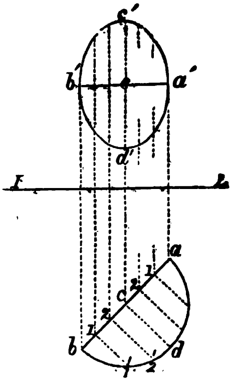
ART. 119.—As a knowledge of the relative position of the several points in the above projections of a circle will be of service in the next chapter, it is necessary to observe that point 1′, No. 3, is the nearest point to the eye, and that its position in No. 4 will therefore be 1 (Theorem 5). The point beyond 1′, No. 3, is 2′, and its position in the lower plane is 2. Again, point 3′ in the lower plane is the plan of 3; and the point beyond 3′, No. 4, is the plan of point 4, No. 3. The motion supposed to be given to Nos. 1 and 2 (as indicated by the arrow *a*, No. 1), in order to produce Nos. 3 and 4, is evidently left-handed; and this is always the case when the elevation required is that seen in direction of the arrow *b*. If the elevation were taken in the opposite direction, the motion would be right-handed: that is to say, the original figures, Nos. 1 and 2, would be turned in such manner that the edges marked 2 2′ would be nearest the eye in the elevation, No. 3, and farthest from the intersecting line in the plan corresponding to No. 3.

PROBLEM XXVIII.

Given the plan of a circular plate which makes an angle of 90° with the horizontal plane and 45° with the vertical plane, to find the elevation.

ART. 120.—At an angle of 45° with the intersecting line, draw *a b*, Fig. 52, equal in length to the diameter of the circular plate to be projected. Bisect *a b* in *c*, and from each of the three points, *b*, *c*, *a*, draw lines at right angles to I L. Parallel to I L, draw *b′ a′*, cutting the centre line drawn from *c* in *e*. From *e*, as a centre, set off *e c′*, *e d′*, each equal to the radius *c a* or *c b*. Then will *b′ a′*, *c′ d′*, be the projection of four points in the figure.

Fig. 52.

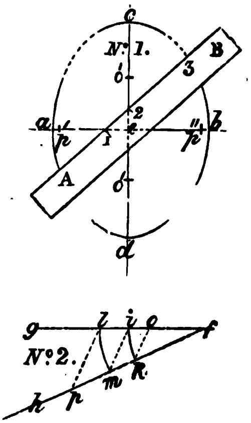


ART. 121.—The projection of any number of intermediate points may be obtained in the following manner — From *c*, as a centre, describe the arc *b d a*, which may be supposed to represent the lower half of the circular plate, bent at right angles to *b a*, the upper half. From *c* draw *c d*, at right angles to *b a*; and parallel to *c d* draw any number of ordinates, 1 1′, 2 2′. Now, since the distance from *c* to the point beyond it is equal to twice *c d*, the radius, it follows that the distance from 2 to the point beyond is equal to twice 2 2′; and so on with points 1 1′. Therefore, from points 1 and 2 in the line *a b*, draw lines parallel to *c′ d′*, making them equal in length, on each side of *b′ a′*, to the ordinates from which they are drawn; and through the points so found draw the line *a′ c′ b′ d′*, which will form an ellipse,—*b′ a′* being what is called the *minor axis* thereof, and *c′ d′* the *major axis*.

ART. 122.—Amongst various methods of describing an ellipse which have been given by different authors, the following has been adopted by the writer for many years in preference to all others; and as there will be several examples in which it may be usefully applied, no apology need be offered for introducing it here.

Let *a b*, *c d*, No. 1, represent the minor and major axes, or the four points through which it is required to describe an ellipse. In any convenient part of the sheet of paper on which the drawing is to be made, draw two lines, *f g*, *f h*, No. 2, at any given angle with each other. From *f*, with the distance *e a*, or *e b*, the semi-axis minor, describe an arc cutting the two lines *f g*, *f h*, in *i k*; and from *f*, with the distance *e c* or *e d*, the semi-axis major, describe an arc, *l m*. Join *i m*; and from *l* and *k* draw lines parallel to *i m*, cutting *f g*, *f h*, in *o* and *p*. From *c* and *d*, No. 1, set off the distance *f o*, No. 2, in points *o′ o′*; and from *o′ o′*, as centres, with *f o* as radius, describe an arc of about 15° on each side of the major axis. From *a* and *b*, No. 1, set off on the minor axial line the distance *f p*, No. 2, in points *p′ p″*, No. 1; and from *p′ p″*, as centres, describe arcs through *a* and *b* of about 15° on each side of the axis *a b**. To obtain any number of intermediate points, take a slip of paper, A B, and mark upon one edge, with a sharp-pointed pencil, 1, 3, equal to the semi-axis major, and 2 3, equal to the semi-axis minor. If the slip of paper be now applied to No. 1, and moved over the figure in such manner that point 2 is always in contact with the major axis, and point 1 in contact with the minor axis, the outer point 3 will describe a perfect ellipse, any number of points in which can be marked off

Fig. 53.



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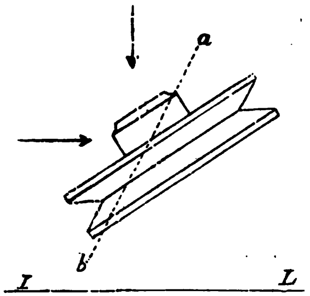
* The length of the arcs on each side of the axis will vary from [illegible] to 40°,—the extent being determined by a correct eye.

as the "trammel," A B, is moved into successive positions. For drawing lines through the points so found, a "French curve" will be of service.*

ART. 123.—As an example for practice the student may now take the following illustration, Fig. 54, which represents an edge view or elevation of a V-grooved pulley, say 5 inches in diameter, with a "boss" in the centre 2 inches diameter. Required a plan of the pulley, which makes any given angle with the intersecting line, also a sectional elevation taken through the line *a b*, and a plan of the section. The ellipses in this example are to be worked out according to the instructions given in ART. 122; and the several points in the line of section are to be obtained as described in ART. 121.

Fig. 54.



PROBLEM XXIX.

To find the projections in the upper and lower planes of a circle which makes any given angle with those planes.

ART. 124.—Let No. 1, Fig. 55, represent a plan of the circle required to be projected, and *a′ b′*, No. 2, the angle which the projected circle is to make with the vertical and horizontal planes. Make *a′ b′*, No. 2, equal to the diameter *a b*, No. 1; and bisect *a′ b′* in *c′*: then will *c′* be the position of *c*, No. 1, and the point beyond *c′* the position of *d*, No. 1 (ART. 92). The projection of the points *a′*, *b′*, *c′*, and the point beyond *c′*, by Theorem 3, will give the minor and major axes of an ellipse. Now measure the distances of points 2 and 4 from the centre line *d c*, No. 1, and set them off on each side of *c*′, No. 2, in points 4, 2. If lines be drawn from 4, 2, No. 2, and from 1, 2, or 3, 4, No. 1, the intersections will give the projection of four intermediate points 1′, 2′, 3′, 4′, No. 3, through which the curve of the ellipse can be drawn. In finding the projection of No. 2 in the lower plane, we may take point *c*′ and the point beyond it to represent points *a* and *b*, No. 1; in which case *a*′ *b*′, No. 2, would correspond to *d c*, No. 1, the projection whereof, together with the intermediate points 1, 2, 3, 4, can be obtained by Theorem 3, as clearly shown by the projecting rays. The above conditions are only true, however, when the original circle is divided into an equal number of equal parts; because, when getting the projection No. 3, No. 2 is supposed to be an elevation of No. 1 looking in the direction *c d;* and when getting the projection No. 4, No. 2 is supposed to be an elevation of No. 1 as seen in the direction *a b:* therefore the four projections cannot be said to agree, although the course here pointed out may in practice be adopted with equilateral and equiangular figures (Art. 98).

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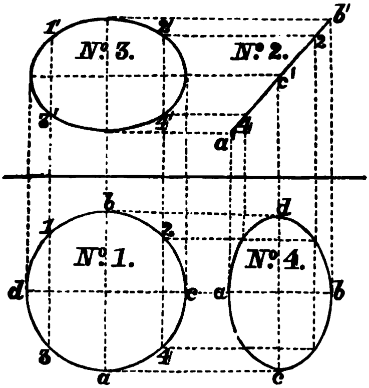
* These curves may be had in great variety of most artists' colormen

Fig. 55.



Problem XXX.

To find in the upper and lower planes the projections of a sphere with twelve meridians and four parallels on each side of the equator.

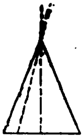
Art. 125.—The projections of a sphere, whether on the vertical, horizontal, or inclined planes, will in all cases be a circle equal in diameter to the diameter of the sphere: such a circle is therefore called the great circle of the sphere.

Let No. 1, Drawing I, represent a plan, and No. 2 the elevation, of a sphere, the axis whereof makes any given angle, *a c d*, with the vertical plane. Divide the circumference of No. 1 into twelve equal parts, and draw the lines A B, C D, E F, &c. Through the centre of No. 2 draw A′ B′ at right angles to *a b*. Divide *a* A′, A′ *b*, each into five equal parts in points 1, 2, 3, 4; and draw 1 1′, 2 2′, &c., parallel to A′ B′. Then will A′ B′ represent the equatorial circle, and the lines 4 4′, 3 3′, &c., the four parallel circles in the upper or northern hemisphere. From the centre of No. 1 describe the concentric circles 1, 2, 3, 4, equal in diameter to the right lines 1 1′, 2 2′, 3 3′, 4 4′, No. 2, respectively. Having now got the plan and elevation of a number of circles, their projections, or the projections of such portions as are visible within the great circle, No. 3, will be obtained according to instructions given in the last problem. In constructing the ellipses the mode described in ART. 122 will be found the best, inasmuch as it avoids the use of construction lines within the figure. The projection of the parallel circles being completed, the twelve meridians, or such portions of them as come within the great circle, No. 3, will be obtained simply by finding the projection of those points in No. 1 where the meridians cut the parallel circles, as 1, 2, 3, 4, G, by Problem XXIX. [See Nos. 1 and 4, Fig. 55]. Through the points so found draw the meridional circles, which will complete the projection of No. 3. The projection of No. 4 will also be obtained by Problem XXIX. [see Nos. 1, 2, and 4], and as clearly shown by construction lines drawn from the parallel circle 3 3′ and meridian G H.

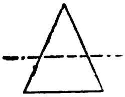
CHAPTER VIII.

ON THE PROJECTION OF CONIC SECTIONS.

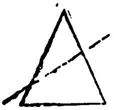
Before entering upon the subject of the projection of conic sections, it is necessary to direct attention to the following definitions:—



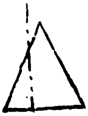
ART. 126.—1. If a cone be cut by a plane passing through the vertex to the base, and making any angle therewith, the true form of section will be an isosceles triangle.*



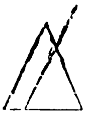
2. If a cone be cut by a plane at right angles to the axis, or parallel to the base, the section will be a circle.



3. If a cone be cut by a plane which passes through the opposite sides and makes an angle with the axis other than a right angle, the section will be an ellipse.



4. If a cone be cut by a plane perpendicular to the base, but not through the axis, the section will be an hyperbola.



5. If a cone be cut by a plane parallel to one of its sides, the section will be a parabola.

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* The true form of section of an object is that which it presents when viewed at right angles to the plane of section.

6. The plan of a cone, the axis of which is perpendicular to the plane of projection, will be a circle, the diameter whereof is equal to the base of the cone.

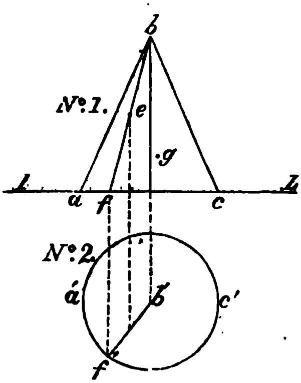
As the delineation of conic sections depends on a knowledge of the projection of a given point upon the surface of the cone, it is presumed that if the following illustration be understood, the geometrical construction of the sections of a cone, in whatever direction it may be viewed, will be a comparatively easy task.

PROBLEM XXXI.

Given the elevation of a cone and the position of a point on its surface, to find the projection of the cone and point in the lower plane.

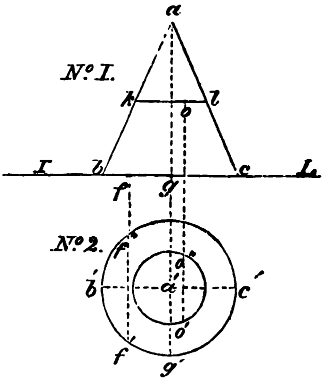
ART. 127.—1st Method. Let $a\, b\, c$, No. 1, Fig 56, be the elevation of a cone, and e the given point. From b, the vertex, draw $b\, b'$ at right angles to the intersecting line; and from any point in the line $b\, b'$, as b', describe a circle, $a'\, c'$, equal in diameter to the base of the cone. Then will $a'\, c'$ be a plan of the cone $a\, b\, c$ (Def. 6). From b draw a right line through the given point e, cutting the base of the cone in f. From f draw $f\, f'$ parallel to $b\, b'$, cutting the circle $a'\, c'$, No. 2. in f'. Join $f'\, b'$. Then will $f'\, b'$ be a plan of the right line $b\, f$, No. 1. If from e, the given point, a line be drawn parallel to $b\, b'$, its intersection with the line $f'\, b'$ will give the plan or projection of the original point. If, however, the given point were at or near the centre of the cone, as g, its position in the plan could not be found

Fig. 56.



as above described. It is, therefore, desirable to adopt that method of finding the projection of a point which is the most general in its application.

Fig. 57.



ART. 128.—2nd Method. Let *b a c*, Fig. 57, represent the elevation of a cone, and b' c' the plan thereof, as before. Through the centre of the plan draw the line b' c' parallel to the intersecting line. Then will the semicircle b' g' c', No. 2, be a plan of the right line *b c*, No. 1: therefore *b c* is the elevation of the semicircle. Again, the plan of any given point or points, as *f* and *g*, No. 1, will be determined by drawing lines therefrom at right angles to the base of the cone, cutting No. 2 in points f' g'; moreover, from *f*, No. 1, to the point beyond, will be equal to f' f'' (ART. 121). This method of obtaining the projection of a point on the base of a cone suggests a simple mode of finding the projection of any given point on the surface of a cone, or the length of any right line drawn through the cone at right angles or perpendicular to the plane of projection. For let *o* be the given point, and let it be required to find its position on the plan, and the length of the line drawn through the cone at that point. Through *o* draw *k l* parallel to the base. *We may now suppose k a l to be a separate and distinct cone, of which k l is the base.* If, therefore, from a', No. 2, as a centre, we describe a circle equal in diameter to the line *k l*, the plan of any given point in that line, and the distance through the cone at that point, will be determined as hereinbefore described for point *f*: that is, the plan of point *o* will be o', and from *o* to the point beyond it will be equal to o' o''.

It will also be manifest that the elevation of any given point, *o′*, on the plan of a cone can be found by reversing the operation; *i. e.*, from *a′* through the given point *o′* describe a circle; find the elevation of the circle on the surface of the cone, as *k l;* erect a vertical line from the given point, as *o′ o;* and the point of intersection with the line *k l* will be the elevation of the given point *o′*.

PROBLEM XXXII.

Given the elevation of a cone and the direction of the line of section; required the plan thereof, a sectional elevation at right angles to the line of section, and also a plan of the sectional elevation.

ART. 129.—Let *b a c*, No. 1, Drawing J, be the elevation of a cone, and A B the line of section. Produce *a d*, the axis of the cone, indefinitely; and from any point therein, as *a′*, describe a circle, *b′ c′*, equal in diameter to the base of the cone. Upon the line of section A B, take any number of points, 1, 2, 3, &c., and find the thickness of the cone at those points; or, in other words, get a sectional plan of the cone *b a c*, by ascertaining the length of the ordinates at points 2, 3, 4, 5 (ART. 128). Find also the plan of points 1 and 6 on the line *b′ c′*, No. 2; and through the points so found draw the curve, which will be an ellipse, or sectional plan of No. 1, as seen from above, perpendicularly to the base of the cone.

ART. 130.—It is required to find a sectional elevation of No. 1, supposing that figure to be viewed at right angles to the line of section, as indicated by the arrow *e*. Draw C D, No. 3, perpendicular to I L; and let C D represent the centre line of the object when projected. From B, the point where A B cuts I L, draw B E parallel to C D. From B as a centre, with the distance 1, 2, 3, &c., as radii, describe the arcs 1 1, 2 2, &c., cutting B E in points 1, 2, 3, &c. Then will B 6, on the line B E, be equal to B 6 on the line of section; the remaining distances will also be equal. From points 1, 2, 3, &c., on B E, draw lines parallel to I L, cutting C D in corresponding points; and make the length of the lines on each side of C D equal to the thickness of the cone at the point from which the line proceeds. For instance, the thickness of the cone at point 3 is equal to 3 3′, No. 2; therefore measure 3 3′ from *b′ c′*, No. 2, and set off those distances on each side of C D, No. 3; do the same with the remaining points; and then draw the curve, which will be an ellipse, representing the boundary of the cone on the line of section.

If No. 1 were seen in the direction of the arrow *e*, the appearance presented by the base would be that of an ellipse, because the base of the cone, which is a circle, is viewed obliquely (ART. 124). From *b*, *d*, and *c*, No. 1, draw lines parallel to the visual rays, cutting the line of section A B in *b′ d′ c′*; and find the position of those points on C D, No. 3, in the same manner as described for points 1, 2, 3, &c. Now from *d*, No. 1, to the point beyond, is equal to the diameter of the base of the cone; therefore, having found the projection of points *b′ c′*, as at *b″ c″*, No. 3, and also *d′* drawn from the centre of the base of the cone; make *d″ d″*, No. 3, equal to the diameter of the base. Then will *d″*, *d″*, *b″*, *c″*, represent four points in the ellipse, which may be drawn as directed by ART. 122. If the line drawn from point *b*, No. 1, cuts the line of section within the cone, as in this case, the two ellipses representing the section and base respectively must be joined by two right lines drawn tangentially to the ellipses, as shown at No. 3; but if the angle of A B be such that the line drawn from point *b* falls without the cone—that is, in some point between A and 6, No. 1—the tangential lines in No. 3 will not be required, because the ellipse or projected base of the cone will be above the ellipse or projected section of the cone.

ART. 131.—It is now required to find a plan of No. 3 as seen in the direction of the arrow *f*; and we would again remark that the student should make an attempt to solve each problem before he reads the explanation of the figure. Presuming that this practice has been adopted and that a little difficulty in realising the position of the figure has been experienced, he must ask himself, what is No. 3? The answer is, an elevation of No. 1, looking in the direction of the arrow *e*: that is to say, the plane of section, No. 1, is at right angles to the visual rays; consequently the plane of section in No. 3 is parallel to the plane of projection; a plan of the plane of section will therefore be a right line. Tangential to the minor axis of the ellipse, No. 3, and at right angles to I L, draw lines 3 3, 3′ 3′. At any convenient distance from the intersecting line draw 3 3′, No. 4, parallel thereto. Then will 3 3′, No. 4, be a plan of the plane of section 3 1 3′ 6, No. 3. This will be evident if we suppose the line of section A B, No. 1, with the base of the cone attached, to move on B as a centre until it coincides with the line E B; and when in that position, if we imagine the cone to turn one-fourth of a revolution on E B as an axis, its appearance would be precisely that of No. 3. Now there is a certain distance from point 1, No. 3, to point *c″*, and also from point 6 to *b″*; which distances will be determined by measuring the length of the lines *b b′*, *c c′*, No. 1. Moreover, since the visual rays, when viewing No. 3 in the direction of the arrow *f*, are parallel to the plane of section, the appearance of No. 3 will be precisely that of No. 1, if seen in the direction of the arrow *g*. The base of the cone will therefore appear as an ellipse, the projecting rays of which are represented by dotted lines *b t*, *d t*, *c t*, drawn from *b*, *d*, *c*, parallel to A B; the distance between the projecting ray from *b* and *c* being the minor axis. Therefore, from 3 3′, No. 4, set off on the centre line C D produced, the several distances *c c′*, *d d′*, *b b′*, No. 1, for the minor axis and centre of the ellipse; from *d″ d″*, No. 3, let fall vertical lines for the major axis; and then describe the ellipse as shown at No. 4. From points 3 3′ draw lines tangential to the ellipse; and the plan of No. 3 will be complete.

Art. 132.—If the student have thoroughly understood the projection of a point on the surface of a cone, he should now be able to find the sectional plan or sectional elevation of such a figure when cut by any plane (as shown by the curved line drawn upon the surface of No. 5), in whatever direction that figure may be viewed: that is to say, whether the projecting rays are in the direction of the arrow *a*, *b*, or *c*. Suppose the direction to be as indicated by the arrow *b*, which is parallel to I L, but which makes an angle of 40° with the vertical plane, and that it is required to find the elevation of No. 5 under these circumstances. Take any number of points in the given curved line, and find by Art. 128 a plan thereof, as shown at No. 6. Then, since $\frac{360}{40} = 9$, it is evident that No. 5 will require to be turned upon its axis one-ninth of a revolution. Let No. 7 represent a plan of the cone in that position, in which case we shall have a number of points in the horizontal plane, and points corresponding thereto in the vertical plane, the projection whereof, as shown at No. 8, will be found by Theorem 3, page 53.

It will also be evident that by the use of the inclined plane a sectional elevation of No. 5 can be obtained, when viewed in the direction of the arrows *a* or *c*, whether such arrows be parallel to or at an angle with the vertical plane.

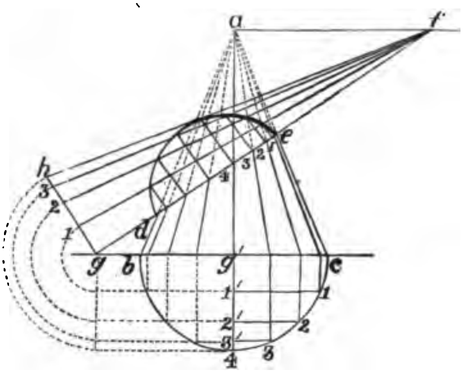
Another mode of obtaining the sections of a cone is given in the next problem.

Problem XXXIII.

Given the line of section which passes through the axis of a cone, to find the section of the cone.

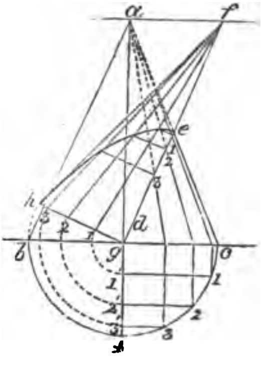
Art. 133.—Let *b a c*, Fig. 58, be the elevation of a cone, and *d e* the line of section. Through the apex of the cone draw *a f* parallel to the base, and produce *d e* both ways, to meet *a f* in *f*, and *c b* in *g*. On *b c* describe a semicircle, and divide it in any number of points, 1, 2, 3, 4. From points 1, 2, 3, 4, draw lines parallel to the base, cutting the axis of the cone produced in points 1′, 2′, 3′. From *g* draw *g h* at right angles to *f g*; make *g h* equal to *g′* 4, and the

Fig. 58.



points in *g h* equal to the points in *g′* 4. From points 1, 2, 3, in *g h*, draw lines to meet in *f*; then from points 1, 2, 3, in the quadrant 4 *c*, draw lines at right angles to the base of the cone; and from the points of intersection with the base, draw lines to the apex of the cone, cutting the line of section *d e* in points 4, 3, 2, 1. If lines be now drawn from the last mentioned points at right angles to *d e*, their intersection with the lines drawn to *f* from corresponding points in the line *g h* will give the form of curve.

Fig. 59.



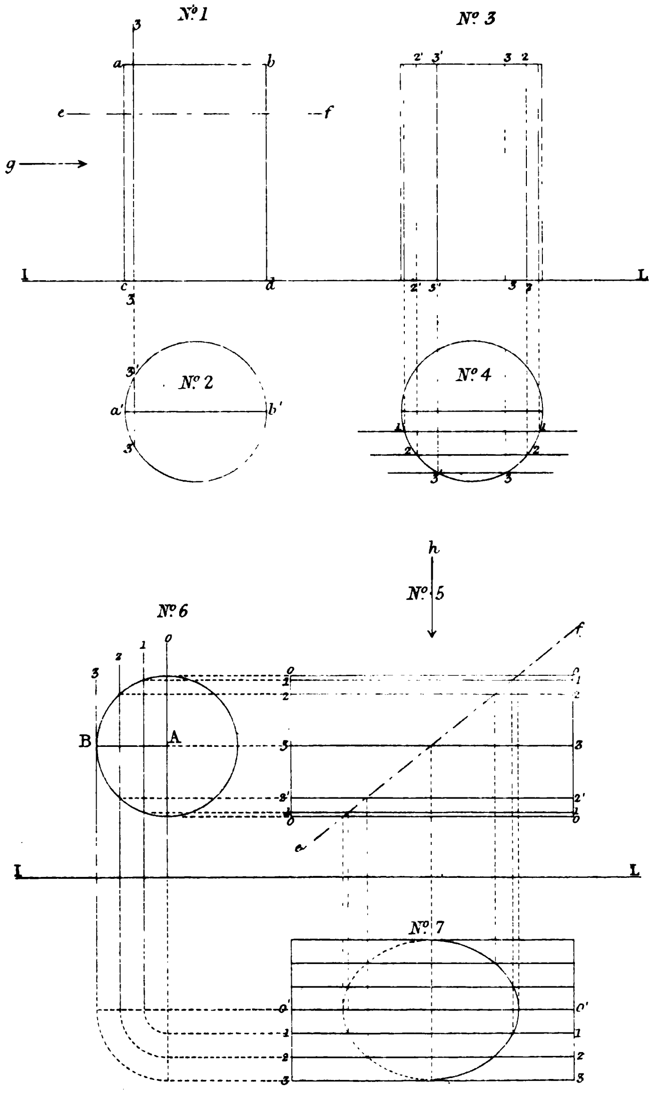
The above figures and letters of reference, with the exception of point 4 on the line of section, apply to Fig. 59, in which the line of section is drawn parallel to the side *a b* of the cone, consequently the form of curve is a *parabola* (Def. 5, page 100).

CHAPTER IX.

ON THE PENETRATION AND INTERSECTION OF SOLIDS.

It sometimes happens in the delineation of the minor parts of mechanical and architectural structures that one solid of revolution, such as a cylinder, cone, or sphere, is joined to or made to penetrate, as it were, another solid. One of the most familiar illustrations is that of a steam or water pipe upon which is cast, at any given angle, a branch pipe of unequal diameter. In this case the joining of the two pipes will produce a line; and it is the projection of such line or lines, caused by the penetration of one solid by another, that we are about to describe. In entering upon this subject, it is necessary to remark that the lines of which the projections are required are found by the use of a number of intersecting planes; that is, by dividing each solid by a plane or planes common to both. These planes may either be perpendicular, parallel, or inclined to the planes of projection; but in order that the operation of finding the projection of the lines of penetration may be as easy as possible, it is desirable, before commencing each problem, to fix upon *that direction for the intersecting plane which will give the most simple form of section.* For instance, a cylinder may be cut by a plane obliquely to the axis; but a more simple form of section would be at right angles or parallel to the axis, as will appear from the two following problems.

PLATE 6.



Wm. Binns delt.

PROBLEM XXXIV.

To find the sections of a cylinder when cut by planes at right angles to its axis, and also when cut by planes parallel to its axis.

ART. 134.—Let *a*, *c*, *d*, *b*, No. 1, Plate 6, represent the elevation of a cylinder, cut by a plane, *e f*, at right angles to the plane of projection,—the axis of the cylinder being parallel to the plane of projection. The section of such a figure will evidently be a circle, equal in diameter to the diameter of the cylinder (ART. 118). Therefore the circle No. 2, of which *a′ b′* is the diameter, is a sectional plan of the cylinder *a*, *b*, *d*, *c*, taken through the line *e f*.*

Suppose No. 1 to be cut by a plane, 3 3, at right angles to the plane of projection and parallel to the axis of the cylinder. The form of section in this case will be a rectangle of the same length as the cylinder, and in breadth equal to 3 3′, No. 2. An elevation of No. 1, if seen in the direction of the arrow *g*, will appear as shown at No. 3, in which the sectional part 3′ 3, 3 3′, is made equal in breadth to 3 3′, No. 2. Again, if No. 4, which is a plan of No. 3, be divided by any number of planes, 1, 2, 3, the projections of those points where the cutting planes come in contact with the cylinder will be obtained in No. 3 by drawing lines from 1 1, 2 2, 3 3, parallel to the axis of the cylinder, as shown by the projecting rays. Then will the lines so drawn represent the point or line of contact of each cutting plane with the surface of cylinder, or, in other words, the entrance and exit of each plane; for instance, plane 2 2 may be supposed to enter the side of the cylinder No. 3 at line 2′ 2′, and leave it at line 2 2.

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* It will be sufficient in these examples to consider the projected figures as sections, without denoting them as such by the section lines described in ART. 12.

ART. 135.—We will now take the case of a horizontal cylinder, of which 0 0 0 0, No. 5, Plate 6, represent a front elevation; the axis of the cylinder being parallel to the intersecting line and also to the plane of projection. If we now suppose such a cylinder to be divided or cut through its axis by a plane, 0 0 0 0, parallel to the vertical plane, and by any number of other planes parallel to the first plane, the end elevation of the cylinder and direction of the cutting planes would be correctly represented by lines 0, 1, 2, No. 6, drawn perpendicular to I L. [Read ART. 28.]

For the sake of perspicuity, we shall, in the problems of this chapter, distinguish the cutting or intersecting planes by 0, 1, 2, 3, &c.: viz., zero plane; first plane, or plane 1; second plane, or plane 2; &c. The zero plane will in all cases be that which passes through the axis of the solid; and the last plane will generally be a plane drawn tangential to the solid, and sometimes called the tangential plane. Again, the distances of the planes 1, 2, 3, &c., from the zero plane will always be measured on a line denoted by the letters A B. [See No. 6.]

ART. 136.—Let it be required to find the projections of the lines formed by the cutting or intersecting planes 1, 2, 3, No. 6, on the surfaces of the two cylinders Nos. 5 and 7.

With regard to the projections of the lines on No. 5, the operation is like that already described for No. 3; the only difference being that of drawing horizontal lines instead of vertical lines from the points of intersection of the cutting planes, 1 2 3, with the surface of the cylinder (No. 6). Then will 2 2, No. 5, be the point or line at which the second plane enters the cylinder, and 2′ 2′ the point or line at which such plane leaves the cylinder. Similar remarks may be made respecting the other planes (ART. 134). In order to find the position of the intersecting planes on the surface of No. 7, measure the distances of the several planes from zero on the line A B, No. 6, and set them off from 0′, in the direction 0′ 3, No. 7. From the points of intersection with 0′ 3, draw lines 1 1, 2 2, parallel to 0′ 0′; and the position of the intersecting planes in No. 7 will be determined. Should the student experience any difficulty in comprehending the foregoing illustrations of the intersecting planes, let him take a cylinder of modelling clay or other soft substance, and apply a knife in the direction of the lines 1, 2, 3, and the difficulty will at once be removed.

PROBLEM XXXV.

To find the section of a cylinder cut by a plane obliquely to its axis.

ART. 137.—Let *e f*, No. 5, be the line of section. Set off, on the opposite side of 0′ 0′, No. 7, the distances 1, 2, 3, and draw lines parallel to 0′ 0′. From those points, in lines 1 1, 2 2, 3 3, No. 5, cut by the line of section *e f*, let fall vertical lines to cut the corresponding lines in No. 7; and through the points of intersection draw the curve, which will be an ellipse; and the sectional plan of No. 5, as seen in the direction of the arrow *h*, will thus be obtained.

PROBLEM XXXVI.

Given the plan and elevation of two cylinders of unequal diameter intersecting each other at right angles, to find the projection of the line of penetration.

ART. 138.—Let No. 1, Drawing K, be the elevation, and No. 2 a plan, of two cylinders with their axes at right angles to each other, but parallel to the upper plane of projection. At any point, A, on the centre line of the horizontal cylinder, describe a circle, as shown at No. 3.

which is an end elevation of the penetrating cylinder. Draw lines 0, 1, 2, 3, perpendicular to the intersecting line, to represent the tangential and cutting planes; and find their position in No. 2, as shown by dotted lines, and explained in ART. 136. We can now suppose the two cylinders, as shown at No. 1, to be divided or cut by the planes 0, 1, 2, which are parallel to the axis of each cylinder, and also to the upper plane (ART. 28). On referring to No. 2, it will be observed that the tangential plane, 3, touches or comes in contact with the vertical or large cylinder in point 3′. If, therefore, a line be drawn from 3′ parallel to the axis of the vertical cylinder, its point of intersection 3″ with the centre line of the horizontal cylinder will give the extreme point in the curved line of penetration; for it will be evident that such point cannot extend beyond 3″,—that is, it cannot be nearer to the centre of the vertical cylinder. We will now take the second plane. From those points where plane 2 cuts the circle, No. 3, draw horizontal lines, which will represent a section of the horizontal cylinder cut by plane 2; and if we find a section of the vertical cylinder cut by the same plane, the intersection of the two sectional planes at 2′ 2′ will give two more points in the curve. A similar mode of proceeding is adopted with the planes 1 and 0. [Read Theorem 3.]

If the student carefully examine the dotted lines drawn upon No. 1, which proceed from any one of the cutting planes, it is confidently hoped that he will thoroughly comprehend the principles which govern the projection of the lines of penetration of solids. For example, if we follow the direction of the lines drawn from those points where plane 2 cuts the circle No. 3, we shall find that the section produced by such plane will be a rectangle or oblong, *a c d b*. Again, the section produced by the same plane passing through the vertical cylinder [see No. 2] will be *e g h f*, No. 1. Now the points 2′ 2′, where these sectional planes meet or intersect, will in all cases be in the curved line of penetration: therefore, the first thing to be considered, in all problems of this nature, is the direction in which the cutting planes are to be applied, so as to give the most simple form of section for both solids.

PROBLEM XXXVII.

Given two cylinders of equal diameter intersecting each other at right angles, to find the projection of the lines of penetration when the axis of the horizontal cylinder is parallel to the vertical plane, and also when it makes any given angle with such plane.

ART. 139.—Let No. 4, Drawing K, represent a plan, and No. 5 an elevation, of two cylinders intersecting each other at right angles, like those parts of gas or water pipes to which the branch pipes are connected. If we adopt the same mode of proceeding as that described for the projection of No. 1, it will be found that the lines of penetration will be projected into right lines as shown at No. 5, instead of curved lines as in No. 1: this is always the case when the two cylinders are of the same diameter.

In finding the projection of these figures, the operation will be simplified if each of the semicircles 0 3 0, 0′ 3 0′, Nos. 4, 5, and 6, be divided into an equal number of *equal* parts, through which lines 1 1, 2 2, No. 5, may be drawn to represent the intersecting planes. Presuming these planes to be drawn in the plan No. 4, and the projection of the lines of penetration to be found as directed in Problem XXXVI., we can now proceed with the projection of No. 6, as shown at No. 7.

Let L S T, No. 6, represent the angle which the axis of the horizontal cylinder is to make with the vertical plane. Produce S T indefinitely; and upon the line so produced, as a centre line, draw No. 6, making it a facsimile of No. 4. From point 3, No. 5, draw a line parallel to I L, as a centre line for the horizontal cylinder No. 7. Find the projection of the end of the cylinder, *a b*, No. 6, and also of the concentric circle representing the inner surface of the cylinder, by ART. 120. Find in like manner the elevation of the semi-cylinder or ellipse c' b', No. 6, in points c'', b'', d'', No. 7. Join c' c'', d' d'', which will complete the projection of the horizontal cylinder, the axis whereof will make an angle with the vertical plane equal to L S T. Through the centre of the vertical cylinder, No. 6, draw f g parallel to I L: then will the semicircle f 3 g be that portion of the cylinder which is presented to the eye. Find the elevation thereof and proceed with the projection of the curves by Theorem 3; for, since points 0′, 1′, 2′, 3′, on the line of penetration, No. 5, correspond with 0′, 1, 2, 3, No. 6, the projection of those points will be found as described. It is necessary, however, to give some explanation about the beginning and termination of the two curves, to remove the difficulties which are often experienced by students when finding their projection. From 0′, No. 6, draw a line at right angles to I L, cutting c' c'' in 0″: then will c' 0″, No. 7, be the elevation of the right line c 0′, No. 6; and the end of the line, or point 0″, will be the beginning of the curve. To find the termination, it will be requisite to determine the length of the right line at point g, No. 6, on the vertical cylinder,—that is, the point at which a right line drawn upon the vertical cylinder at g would come in contact with the horizontal cylinder. Measure the chord of the arc from 3 to g No. 6, and set off the length of the chord from 3 to g No. 4. From g, No. 4, draw g' h, No. 5, parallel to 3 3′: then will g' h be the length of the right line, and h the point at which that right line will come in contact with the horizontal cylinder. From h, No. 5, draw a line parallel to I L, cutting the vertical line drawn from g, No. 6, in h', which will be the termination of the curve.

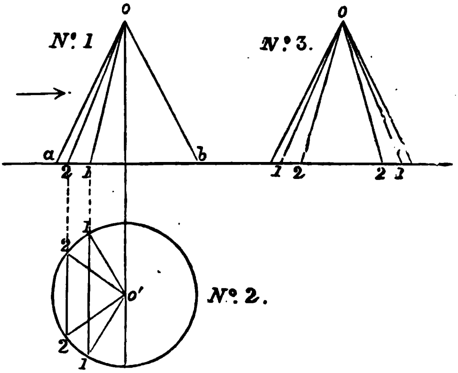
ART. 140.—The style of shading exhibited in No. 7 is frequently resorted to in outline drawings of machinery, for the purpose of producing the effect of a cylindrical surface. The operation is as follows:—Having deter mined the darkest part of the cylinder (ART. 64), commence at that part by drawing thick lines parallel to the axis, taking care that each successive line is somewhat lighter or thinner than the first, which may be effected by adjusting the screw of the drawing pen, or going over the line a sufficient number of times to produce the desired effect; observe also that the distance between each line is gradually increased in both directions from *n* (the darkest part) so as to produce on the left of *n* what is called the reflected light. To ensure success, practice and good judgment in the distribution and thickness of the lines will be required.

PROBLEM XXXVIII.

To find the projection of a cone penetrated by a cylinder, the two axes being at right angles to each other but parallel to the vertical plane of projection.

ART. 141.—If a cone, as *a o b*, Fig. 60, were cut by a series of planes from the vertex to the base, each section would be an isosceles triangle [See Definitions, page 100]. Now let No. 1 be an elevation, and No. 2 the plan of a cone cut by planes o 1, o 2. From 1, 2, No. 1, draw vertical lines cutting the base of the cone No. 2 in points 1 1, 2 2; and from the points of intersection with the circle representing the base, draw lines to the centre o′: then will 1 1 o′, 2 2 o′,

Fig. 60



be a plan of the intersections of the cone cut by planes O 1, O 2, No. 1. Again, let No. 3 be an elevation of the cone, No. 1, as seen in the direction of the arrow; if 1 1, 2 2, No. 3, be made equal to 1 1, 2 2, No. 2, and lines be drawn from those points to the apex, No. 3 will then represent an elevation of the cone and lines of section formed by the cutting planes, each of which will be an isosceles triangle. This, therefore, is one of the simplest forms of conic sections.

Presuming the direction of these cutting planes to be understood, we can now proceed to a solution of the problem, reference being made to Drawing L, in which No. 1 represents the elevation of a cone penetrated by a cylinder, and No. 2 a plan of the base of the cone. From point *n*, No. 1, with the same radius as the cylinder, describe a semicircle, which will represent an end elevation of the half or visible portion of the cylinder; and draw O 1, O 2, the cutting planes, and O 3, the tangential plane. From A, No. 2, on the line O A produced, set off A 1, A 2, A 3 (No. 1), in points 1′, 2′, 3′, No. 2; and from these points draw lines parallel to I L, cutting the base of the cone in 1″ 1″, 2″ 2″, 3″ 3″. Find the projection of the last mentioned points on the base of the cone No. 1, and draw lines therefrom to the apex, as shown. If lines be now drawn from those points where O 1, O 2, No. 1, cut the semicircle or semi-base of the cylinder, the intersection of such lines with the lines drawn from the base of the cone will give so many points in the line of penetration: the form of curve produced by the penetrating cylinder will therefore be obtained by drawing lines through the aforesaid points of intersection, as shown at No. 1. The point of contact of the tangential plane with the semicircle may in all cases be determined by drawing from the point *n* a line at right angles to the tangential plane O 3.

The same result, namely, the curved line of penetration, may also be obtained by intersecting planes at right angles to the vertical plane and parallel to the horizontal plane: that is to say, if the two solids were divided by a plane perpendicular to the upper plane and parallel to the intersecting line, the horizontal section of the cone would be a circle; and the same line of section through the cylinder would be a rectangle, the intersection of which with the circle would give the projection of a point in the curved line of penetration.

Problem XXXIX.

Given the plan of a cylinder penetrated by a cone, to find the line of penetration, the axis of the cone being parallel to the horizontal plane, but making any given angle with the vertical plane, and the axis of the cylinder being at right angles to the axis of the cone.

Art. 142.—Let *a b*, No. 3, Drawing L, be a plan of the cylinder penetrated by the cone *e g f*. Draw *a b d c*, the elevation of the cylinder. Parallel to I L draw *e f g′*, No. 4, the axis of the cone. Find by Art. 120 the projection *e h f i*, the base of the cone. From *g′* draw lines *tangential* to the ellipse *h f i*. From *h*, No. 3, as a centre, with the same radius as that of the base of the cone, describe the semicircle *e h′ f*. Divide *e h′ f* into an *even* number of *equal parts*, in points 2, 1, *h′*, &c.; and from the points of division draw lines at right angles to the base *e f*, cutting it in points 2′, 1′, *h*, 1, 2: then will these points be the projections of the corresponding points on the semicircle *e h′ f*. Upon the major axis or diameter of the ellipse *h i*, No. 4, set off the several points taken from the diameter *e f*, No. 3; and draw lines at right angles to *h i*, cutting the ellipse in points 1, 2, *f*, 2, 1. From 1′, 2′, No. 3, draw lines to *g*, the apex of the cone. If we now suppose 1′ *g*, 2′ *g*, to be intersecting planes, cutting the two solids in a direction perpendicular to the lower plane of projection, or parallel to the axis of the cylinder, we shall get the simplest forms of section of which the two solids admit when simultaneously cut by intersecting planes; the section of the cylinder being a rectangle, and that of the cone an isosceles triangle. The points of intersection in the curved line of penetration will be where the corresponding sections meet or intersect each other. It will also be understood that *h*, No. 4, is the projection of point *h*, No. 3; and *f*, No. 4, the projection of *f*, No. 3. Now the line *f*, No. 3, comes in contact with the cylinder at *k*: therefore *k*, No. 4, will be the extreme point in the curve. Again, the section lines $2' g$, $1' g$, No. 3, come in contact with the cylinder at points *l*, *m*; and the projection of these points will be found on the corresponding lines drawn from 1 2, 2 1, to g', No. 4, as shown by projecting rays from *l* and *m*. We have now to find the terminal points in the curve. From *t*, No. 4, the tangential point of contact of the line drawn from g', let fall a vertical line, cutting *e f*, No. 3, in t'; and draw $t' g$. Then will $t' g$ be that portion of the surface of the cone which is seen in the elevation No. 4, and its point of contact with the cylinder and its projection will be found as explained for points *l* and *m*.

PROBLEM XL.

Given the elevation of a cone penetrated by another cone, to find the line of penetration, the axes of the two cones being at right angles to each other but parallel to the plane of projection.

ART. 143.—Let *b a c*, No. 5, Drawing L, represent the elevation of a cone with its axis perpendicular to I L, and *e d f* the elevation of the penetrating cone, with its axis parallel to I L. Through the base of the penetrating cone draw C *f e* indefinitely; and through the apices of the two cones draw the right line *a d*, cutting the prolonged base line of each cone in points O′ O. Upon *e f* and *b c* describe a semicircle, with a radius equal to the semidiameter of the base of each cone respectively. From O′ draw O′ 1, O′ 2, the direction of the cutting planes, and O′ 3, the tangential plane. From A, upon the line A C, set off the distances A 1, A 2, A 3, in points 1′, 2′, 3′; and from the points of intersection on the line A C draw 1′ O, 2′ O, 3′ O, cutting the semicircular base of the cone in *r*, *s*, *t*. From *r*, *s*, *t*, draw lines at right angles to the base of the cone; and from the points of intersection on the base *b c* draw lines to the apex of the vertical cone. Proceed in like manner with the horizontal cone: that is, find the projections of the intersecting points of the cutting planes with the semicircular base on *e f;* and draw lines to the apex *d*. Then will the intersections of these lines with the corresponding lines drawn upon the vertical cone give so many points in the curved line of penetration.

ART. 144.—In order that the student may clearly understand the direction of the intersecting planes 1, 2, 3, and the reason for drawing the line *a d* to meet the prolonged base line of each cone in points O O′, and also the lines from points 1′, 2′, 3′, converging to the point O, he is recommended to cut a piece of cardboard to the same form and dimensions as the triangle A O′ O. If this piece be placed flat upon the drawing, so as to coincide with the triangle A O′ O, it will represent the zero intersecting plane, or a plane passing through the axis of the two cones. Let the angle or corner A be raised from the surface of the paper a distance equal to A 1, or A 1′, and supported in that position with the hypothenuse resting on the line O′ O; and the cardboard will then show the exact position of the first intersecting plane. If the angle A be further raised a distance equal to A 2, with the hypothenuse turning on O′ O as a hinge-joint, the cardboard will indicate the position and direction of the second intersecting plane; and if the angle A be raised through a distance equal to A 3, the cardboard will represent a tangential plane to the lesser cone and the third intersecting plane of the greater cone,—the lines of intersection being r' a, s' a. The two cones are therefore simultaneously divided by planes which are common to both, and which give the most simple form of section, *i.e.*, an isosceles triangle.

ART. 145.—If the two cones were divided by planes parallel to the intersecting line, but at right angles to the vertical plane, the sections of the vertical cone would be circles, and the sections of the horizontal cone hyperbolas; —the line of penetration being in points where the two sections meet. This mode of obtaining the curve would, however, be much more difficult than that described. No. 6 represents a plan of the two cones, in which the several points in the line of penetration are obtained by ART. 128.

PROBLEM XLI.

Given the position of a point or line upon a sphere in one of the planes, to find the projection of the point or line on the surface of the sphere in the other plane.

ART. 146.—The plan and elevation of a sphere will be represented by two circles equal in diameter to the diameter of the sphere, as shown at Nos. 1 and 2, Drawing M. Again, if a sphere be cut by any plane, as a b' c, No. 1, and viewed at right angles thereto, the section produced will be a circle.

ART. 147.—Let a, No. 1, represent the position of the point of which it is required to find the plan or horizontal projection. Through a draw a c parallel to I L. From b, No. 2, as a centre, and with b' c, No. 1, as a radius, describe a circle; and from a, the given point, draw a line parallel to the line of centres b' b, cutting the circle in a'. Then will a' be the plan of a. Moreover, a line drawn through the sphere at point a, at right angles to the vertical plane, will be equal in length to a' a'' (ART. 128).

Let *e* be the position of a point on the *lower* hemisphere, and let it be required to find its projection in No. 1. Through *e* draw *g f* at right angles to the line of centres, or parallel to I L; with *f g* as a radius, and from *f*, the centre of No. 1, describe an arc of a circle, *o p*; and draw *e e′* parallel to the line of centres. Then will *e′*, the intersection of the right line with the arc *o p*, be the projection of point *e* on the lower hemisphere.

Inasmuch as the projection of any line drawn upon the surface of a sphere can be obtained by finding the projection of any number of points in that line, it is presumed that the student will experience no difficulty in finding the projection of the curved line *h i*.

ART. 148.—Before leaving this subject, the student is recommended to find the sectional elevation of a sphere cut by a plane perpendicular to the horizontal plane, and making any given angle with the vertical plane. Let *k l* be the line of section of which an elevation is required. If the projection of such a line be obtained as directed for the projection of a point, the elevation will be an ellipse; it will also be found that the length of the right line *k l* between the points of intersection with the great circle of the sphere will be equal to the *major* axis, and that the *minor* axis will be equal to *m n*. The projection of such a figure can therefore be obtained by means of the triangular figure described in ART. 122.

With regard to the projection of Nos. 3 and 5, which represent the plan of a sphere penetrated by a pentagonal prism, and the plan of a sphere penetrated by a cylinder and also perforated by a cone, it will be unnecessary to remark further than that the most simple form of section will be produced by cutting planes drawn parallel to the intersecting line, and that the projection of points 1, 2, 3, 4, on the upper and lower hemisphere, will be found as directed for point *e*, No. 1, and so on for any number of points in the curve or line of penetration.

PROBLEM XLII.

Given the plan of a sphere penetrated by a cone the axis of which is eccentric to the axis of the sphere, to find the line of penetration.

ART. 149.—Let A, No. 7, Drawing M, be the centre of the sphere, and B that of the cone. Draw *e b′ f*, No. 8, the elevation of the cone; and upon the line of centres A *a* draw the great circle *a g h* of the sphere supposed to be penetrated by the cone *e b′ f.* Through the centre A, No. 7, draw A *x* at right angles to A *a;* and from A through B, the centre of the cone, draw A C. From C draw C *c′* parallel to A *a;* and from *c′* draw *c′ b′*. Then will the line *c′ b′* on the cone *e b′ f* be the elevation of the line C B, No. 7.

We will now determine the points at which the line *c′ b′* will enter the lower and leave the upper hemisphere, or, in other words, its entrance into and exit from the sphere. From A as a centre, with A C and A B as radii, describe arcs C *c* and B *b*. If we now suppose the sphere and cone to turn on A as a centre until the line A C coincides with A *x*, the point *b* will represent the position of the apex of the cone, and *c* the position of point C in the base. Find the elevation of point *c* in *c″*, and of point *b* in the line *b′ b″* drawn from *b′*, No. 8, parallel to I L; and join *c″ b″*. Then will *c″ b″* be the elevation of the line B C in its new or assumed position, and *g h* the points at which that line comes in contact with the sphere. If lines be now drawn from *g* and *h* at right angles to the axis of the cone, the intersection of those lines at *i* and *k* with *c′ b′*, the real position or elevation of B C, will give the highest point in the lower curve and the lowest point in the upper curve;—that is to say, every point in the curve on the lower hemisphere will be below *k*, and in the upper hemisphere above *i*. The student has now to find the projection of a number of other points through which the curve is to be drawn; and the most simple form of section for this purpose will be that of a horizontal plane

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drawn parallel to the base of the cone. At any distance below *k h*, draw any number of lines, *l m*, *n o*, to represent the direction of the cutting or intersecting planes, and proceed to find the sectional plan of the cone and sphere at each line of section, which will manifestly be two circles (ARTS. 128, 146); and the elevation of the points of intersection in those circles will give, on the corresponding line of section, the projection of two points in the curve. Thus, the section of the sphere taken through the line *l m* will be a circle equal to *l m*; therefore from A, No. 7, with *l t* or *m t* as radius, describe an arc, *l′ m′*; and from B, the centre of the cone, with a radius equal to the radius of the cone taken on the line *l m*, describe arcs cutting *l′ m′* in *p q*. If lines be now drawn from *p* and *q* parallel to A *a*, their intersection in points *r* and *s* will give the projection of two points in the line of penetration. In like manner any number of points may be found below *k* and above *i*, and the projection of the curve completed.

ART. 150.—It is desirable to remark, with regard to the subject of the penetration of solids, that the student must not be surprised, when finding the projection of objects similar to those we have given, if the form of curve should not be the same as shown in our illustration, for it will be manifest that the character of the curve will be governed by the magnitude and relative position of the two solids. For instance, in order to produce the same curved line of penetration as that shown at No. 7, when drawing from a model of a sphere penetrated by a cone, it will be necessary to measure exactly the diameter of the sphere, the height of the cone and diameter of its base, the distance of its axis from the axis of the sphere (or amount of eccentricity), and the angle which the line A C makes with the intersecting line; all of which conditions must be strictly observed in order to produce the appearance which would be presented by a model of the objects just described in this as well as all other problems relating to the penetrations of solids.

PROBLEM XLIII.

Given the plan of an annulus or ring penetrated by a cylinder, to find the projection of the line of penetration in the vertical plane.

ART. 151.—Let No. 1, Drawing N, represent the plan, and No. 2 the elevation of a ring (having a circle for its common or transverse section), with a cylinder, A B D C, penetrating the ring. If the ring be cut by planes at right angles to the horizontal plane in the direction of *a b*, No. 1, the sectional elevations will be a series of elliptical figures; but if the ring be cut by planes parallel to the horizontal plane, the elevations of these planes will be right lines, and the horizontal projections of the ring cut by such planes will be concentric circles. This direction of the cutting planes will therefore give the most simple form of section for the annulus. Let *c d*, No. 2, be the direction of the intersecting planes. From *e*, No. 1, the centre of the ring, with $e' f$, $e' g$, No. 2, as radii, describe two concentric circles, f', g', No. 1, cutting the cylinder in points 1 and 2. Then will the two circles f', g', represent a sectional plan of the ring taken through the line *c d*. From point 2 draw a line parallel to the line $e\ e'$, cutting *c d* in point $2'$: then will $2'$, No. 2, be the projection of a point in the curved line of penetration. The projection of any number of points may be found in like manner, observing that all those points in the plan which fall beyond the centre line, 1 3, of the cylinder are invisible, and must, if represented, be drawn in dotted lines.

PROBLEM XLIV.

To find the sectional elevation of an annulus or ring.

ART. 152.—Let *h i*, No. 1, Drawing N, be the line of section, and *h i k* that part of the ring supposed to be removed. Having taken any number of points, as 4, 5, 6, 7, 8, we have simply to find the distance from each point to the point beyond it, or the thickness of the ring at those points, the projection of which in the vertical plane will give the form of section. From *e*, No. 1, the centre of the ring, draw *e l* parallel to I L; and let the section of the ring taken through the line *e l* be a circle, as shown by *g* 4″ *f*, No. 2. From *e*, the centre of the ring, with *e* 4, *e* 5, &c., as radii, describe arcs cutting *e l* in points 4′, 5′, 6′. Now the thickness of the ring at point 4 will be found by drawing a line from 4′ parallel to *e e*′, cutting the sectional elevation of the ring in points 4″ *g* (ARTS. 118, 121). Therefore, from point 4, No. 1, draw the line 4 *s t*, parallel to *e e*′; and from points *g* and 4″ draw lines parallel to *c d* or I L, cutting the line drawn from point 4 in *t* and *s*. Then will *t s* be the projection of point 4 and the point beyond 4, or, in other words, the thickness of the ring at that point. The projection of points 5, 6, 7, 8, through which the curve will pass, are found in like manner.

PROBLEM XLV.

Given the projection of a sphere penetrated by a cone, the axis of the cone being at any given angle to the vertical plane, but parallel to the horizontal plane, and at any given distance from the axis of the sphere, to find the line of penetration.

ART. 153.—Let No. 3, Drawing N, represent an elevation, and No. 4 the plan of a sphere penetrated by a cone. Draw *a b*, No. 4, the axis of the cone, at any given angle to I L, and *a*′ *b*′, the axis of the cone No. 3, parallel to I L, and at any given distance from *c d*, the axis of the sphere. From *a*, No. 4, as a centre, with *a f* as radius, describe an arc or quadrant, *f n*, and divide it into any number of *equal* parts in points 1, 2, 3. Find by

Problem XXXIX. the projection of points 1, 2, 3, on the base of the two cones, Nos. 3 and 4; and draw lines therefrom to the apex of each cone. Then will 1′ *b*, 2′ *b*, &c., No. 4, represent the direction of the intersecting planes, which are perpendicular to the plane of projection. It will also be seen that *g*, *b*′, *h*, No. 4, the sectional elevation of the cone taken through the line 2′ *b*, will be an isosceles triangle, and that the sectional elevation of the sphere taken through the same line of section, from *i* to *k*, will be an ellipse [read Art. 148]. Find the projection of those two points where the ellipse cuts or intersects the isosceles triangle; and two points in the line of penetration of a sphere by a cone will thus be obtained. Proceed in like manner with the other sections; care being taken to find, *and work from*, the major and minor axes of each ellipse.*

PROBLEM XLVI.

Given the plan and elevation of a cone intersected by a sphere, to find the contour of the concavity on the surface of the cone in the upper and lower planes.

Art. 154.—Let No. 5, Drawing N, represent the elevation, and No. 6, a plan of a cone intersected by a sphere. From *a*, the axis of the cone No. 6, and through *b*, the centre of the sphere, draw *a b c:* then will *a c* represent a plan or edge view of a great circle of the sphere coincident with a right line, *a f*, drawn on the surface of the cone. From *f*, draw *f f*′, parallel to the line of centres *a*′ *a;* and draw a line from *f*′ to *a*′, No. 5, which will be the elevation of the right line *f a*, No. 6. If we now find a sectional elevation of the sphere taken through the line *a c*, the intersection of that figure with the right line *a′ f′* will give the highest and lowest points in the contour of the curve. These points may, however, be more readily found by supposing the sphere and cone to turn upon *a*, as a centre, until the line *a c* coincides with *d e*. From *a*, as a centre, with *a b* as radius, describe an arc cutting *d e* in point *g*; from *g* draw *g g′* parallel to *a′ a*; and from *b′*, the centre of the sphere, No. 5, draw *b′ c′* parallel to I L, cutting the line drawn from *g*, No. 6, in *g′*, No. 5. Then will *g′* be the new or assumed position of the centre of the sphere. From *g′*, with a radius equal to that of the sphere, describe the arc *h c′ i*, cutting the side *a′ k* of the cone in points *h* and *i*; from *i* and *h* draw lines parallel to I L; and the intersection of these lines with *a′ f′* will give the projection of the highest and lowest points in the curve. If, in this example, the intersecting planes are drawn at right angles to the vertical plane and parallel to the horizontal plane, as shown by the dotted line *l m*, the simplest form of section—namely, a circle—will be produced in both figures. Therefore, at any distance below the highest point, draw *l m* parallel to I L; find the sectional plan of the sphere and cone; and their intersection will give two points in the curve, as in Problem XLII.

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* Since two points only in each ellipse are required, the "trammel," described in Art. 122, will be the most expeditious means of obtaining these points.

Although this interesting part of our subject might be greatly extended, it is presumed that the foregoing examples, if carefully worked out and studied, will be sufficient to impress upon the mind of the student that the projection of the lines of penetration of solids must be obtained by the use of intersecting planes, and that the easiest solution of any problem is to apply those planes in such manner as to give the most simple form of section. It will be manifest, however, that such projections cannot be obtained without a previous knowledge of the mode of obtaining horizontal and vertical sections of the solids separately considered: these we have already explained. We have also shown that the position of any single point in the line of penetration will, in all cases, be where the two sections produced by the same intersecting plane meet each other. Here, then, we have a reason for the projection of every point, and for the form of curve produced. Should the student fail, however, in realising the exact form of the curve after its projection has been obtained by the rules which we have given, we should recommend him to take one of the "Darmstadt" models* and lay down its dimensions as directed in ART. 150, when he will find that his work is a perfect fac-simile of the model before him.

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* These beautiful models of the penetrations of solids, which are used on the Continent for teaching mechanical drawing, are now added to the collection of educational models manufactured at the Chester Training College by Messrs. Arthur and James Rigg, and may be seen in the Educational Museum at South Kensington.

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CHAPTER X.

On the Development of the Curved Surfaces of Cylinders and Cones.

The curved surface of a solid upon which a flexible but non-elastic material, such as a sheet of paper, can be bent so as to apply evenly and without vacuities, is said to be a developable surface; and the curved surface of a solid upon which a flexible material cannot be bent without leaving vacuities, is said to be a non-developable surface. A sheet of paper or other flexible material which, when bent, is capable of covering and coinciding with every part of the surface of a solid, is called the envelope of that surface.

Problem XLVII.

To find the envelope of the curved surface of a semi-cylinder.

Art. 155.—Let *a b d c*, No. 1, Plate 7, be the elevation of a semi-cylinder. Upon *c d*, the diameter, describe the semicircle *c e d*, which will be a plan of the semi-cylinder. Draw *a f* and *c g* at right angles to *a c*, and equal in length to the arc *c e d;* draw *f g* parallel to *a c;* and *f a c g* will be the envelope of the semi-cylinder *a b d c*.

Art. 156.—As there is no correct geometrical method by which an arc can be made equal in length to another arc described with a different radius, or a right line equal in length to a given arc, the most convenient way is to divide the given arc by a pair of compasses into a number of equal parts, and then, without altering the adjustment of the compasses, to set off on the right line the same number of parts; but as the chord is always less than the arc, the right line *c g* will be something less than the curved line *c e d*. Notwithstanding this discrepancy, if the arc be divided into small portions, the result will be sufficiently near for our purpose. In cases where greater accuracy is required, the following method may be adopted: Multiply the diameter of the circle by 3·1416, and the product will be the circumference. Divide this product by 2, and the quotient will be the length of the semicircle.

PROBLEM XLVIII.

Given the position of a point on the envelope of a semi-cylinder, to find its projection on the curved surface thereof.

ART. 157.—In this proposition it is required to find the projection of a given point on the surface of a semi-cylinder, supposing the sheet of paper to be wrapped around the cylinder; in other words, *h*, No. 1, being the given point of *development*, it is required to find the projection of the point of *envelopment*. From *h*, draw *h i* at right angles to the axis of the cylinder, cutting *a c* in *k*. Divide *h k* into equal parts; and upon the semicircle *c e d*, with the length of one of the parts, set off the same number of parts, from *c* to *e*. Draw *e i* parallel to *d b*; and its intersection in *i* with the line drawn from *h* will be the projection of the given point *h*.

PROBLEM XLIX.

Given the development of a right line on the envelope of a cylindrical surface, to find the projection of the line upon that surface.

ART. 158.—Let *c h*, No. 2, be the given right line. Divide the arc *c e d* and the line *c a* into the same number

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of equal parts in points 1, 2, 3, 4, and 1″, 2″, 3″, 4″; and from points 1′, 2′, 3′, 4′, on the line *c g*, draw lines parallel to *a c*, cutting *c h* in points *i*, *k*, *l*. Find the projection of *i*, *k*, *l*, as explained for the projection of a point (Problem XLVIII). Through the points of envelopment *i′*, *k′*, *l′*, draw the curve *c i′ k′ l′ h′*, which will be the projection of the right line *c h* on the curved surface of the cylinder *a b d c*.

In like manner the development of a right line drawn on the curved surface of a cylinder, as *c h*, No. 3, can be found by dividing *c g* and *c e d* into the same number of parts; drawing lines from 1, 2, 3, in the curved line *c e d*, cutting the given right line in points *i*, *k*, *l*; and then finding the development of those points on the envelope, as shown by the lines of construction.

No. 4 shows the development of the curved surface of a cylinder perforated with a cylindrical opening, and, consequently, the form of opening that would be required in the envelope to coincide with that of the cylinder. In projections of this kind it will be evident that greater accuracy will be ensured by increasing the number of points. As the principles contained in these propositions are employed in the projection of screws, and also in the projection of the well and geometrical staircases, the student is recommended to work out No. 2 very carefully for an entire revolution of the cylinder, and to pay attention to the form of the curve on its return at points *c* and *h′*, which will be of the same character above the point *h′*, or right line *h h′*, as below.

PROBLEM L.

To find the envelope of the curved surface of a semi-cone.

ART. 159.—Let *b a c*, No. 1, Plate 8, be the elevation of a semi-cone, and *b d c*, a semicircle equal to the base. From *a*, with *a b* as radius, describe the arc *b e*; and make *b e* equal to the arc *b d c*. Join *e a*; and *e a b* will be the envelope required.

PROBLEM LI.

Given the projection of a line on the curved surface of a semi-cone, to find the development of the line.

ART. 160.—Let *b a c*, No. 2, be the semi-cone; *a e c*, the envelope; and *a f* the given right line. Draw *f g* at right angles to the base *b c*; and make *c h*, on the arc *c e*, equal to the arc *c g*. Join *h a*; and *a h* will be the development of the line *a f*.

PROBLEM LII.

Given the projection of a point on the curved surface of a semi-cone, to find the development thereof.

ART. 161.—Through *i*, No. 2, the given point, draw *a k*. From *k* draw *k d* at right angles to the base *b c*; and from *i* draw *i l* parallel to the base, cutting *a c* in *l*. Make *c m*, on the arc *c e*, equal to the arc *c g d*; join *m a*; and from *a*, with the radius *a l*, describe an arc cutting *a m* in *n*. Then will *n* be the development of the given point *i*. The position of *n* on the line *m a* may also be found by drawing a line from the given point *i* parallel to *a b*, cutting the base in *o*, and setting off from *m* on *m a* the length of the line *o i*, which will give the development of the given point *i* in *n*.

PROBLEM LIII.

Given the dimensions of a lamp-shade or reflector, to find the envelope of its surface.

ART. 162.—Draw *a b*, No. 3, the centre line. Make *b c* equal to the vertical height of the lamp-shade; *d e* equal to the diameter at the base; and *f g* equal to the diameter at the top. Through *f* and *g*, draw *d a*, *e a*. From *a* as a centre, with *a f*, *a d* as radii, describe the arcs *d h*, *f i*. Make *d h* equal to the *circumference* of the base *d e* (ART. 156.); and join *h a*. Then will *d h i f* be the envelope or covering for the lamp-shade,—allowance being made for the lap or joining, as shown by dotted lines.

PROBLEM LIV.

To find the development of a line situated on the curved surface of a cone.

ART. 163.—Let *b a c*, No. 4, be the cone; *e a b*, the envelope; and *b f* the given line. Divide the arcs *b d c* and *b e* into the same number of equal parts in points 1, 2, 3; 1′, 2′, 3′. From the points of division on the arc *b d c*, draw lines at *right angles* to the base of the cone; and from the points of intersection on the base draw lines to *a*, the vertex of the cone, cutting the line *b f* in points *g*, *h*, *i*, &c. Find, by Problem LII., the position of the points *g*, *h*, *i*, &c., on the envelope; and draw the curve *b f′*, which will be the development of the right line *b f*.

If the student will read ART. 109, he will there find that all right lines drawn from the apex of a cone to the base (termed the meridians of the cone) are of equal length; and since all right lines drawn from the centre of a circle to the circumference are of equal length, it is manifest that the actual length of the lines *a* 2, *a* 3, *a* 4, &c., is equal to the length of the lines *a* 1′, *a* 2′, *a* 3′. It follows, therefore, that if the envelope *e a b* were applied to the surface of the cone, the meridians and their developments would coincide: consequently the curved line *b e* would coincide with the right line *b c* or base of the cone. For a practical illustration of this subject, the student is recommended to construct No. 3 on a loose sheet of paper, and with a sharp penknife to cut out the envelope, which, when folded, will produce the frustrum of the cone *d f g e*. The same experiment may also be tried with the other figures upon which meridian lines have been drawn.

PROBLEM LV.

To find the projection of a proportional spiral on the surface of a cone.

ART. 164.—Let *b a c*, No. 5, represent the cone, and *a e c* the envelope upon which the meridian lines and their development have been drawn. Upon the envelope *a e c* draw the curve or proportional spiral *c d e f h*, making equal angles with the radii *a* 2, *a* 3, &c. To effect this, *a d c being the given angle*, adjust a pair of proportional compasses in such manner that when the long legs are opened to the extent of *a c*, the distance between the points of the short legs will be equal to *a d*. If the distance *a d* be now taken with the long legs, the short legs will give the proportional length *a e;* and so on with the remaining points. From *a* as a centre, with *a d, a e, a f*, &c., as radii, describe arcs cutting *a c* in *g, h, i;* and from the points of intersection *g, h, i*, draw lines parallel to the base of the cone, cutting the meridian lines corresponding to those of the envelope in points *k, l, m*, which will give the direction of the curve for half a revolution of the spiral. If the projection of the spiral is required to be continued, repeat the operation just described upon each meridian; and every repetition will give the development of a new line, the projection whereof will be the projection of another half revolution of the spiral. The first point of the second curve on the line *a b* will be the same distance from the vertex *a* as *a o* or *a h*, the last point in the curve *c d e f h*. The drawing before us shows two revolutions of the spiral, which, however far continued, would never terminate in the vertex *a*.

CHAPTER XI.

EXAMPLES IN PRACTICE.

It is proposed under this head to exhibit an application of the principles set forth in Problem XLIX. to the projection of screws.

ART. 165.—The thread of a screw, whether of the square or V form, is a helix wound upon a cylinder; and its development upon the envelope of the cylinder will be a right line (ART. 158). If the envelope be that of an entire cylinder, the inclination of the right line, or the angle which it makes with the axis of the screw, will represent what is called the pitch of the screw.*

ART. 166.—Let the right line *a b*, No. 1, Drawing O, be made equal in length to the circumference of the screw. From *a* draw *a c* at right angles to *a b*; and make *a c* equal to the pitch of the screw. If a right line be now drawn from *c* to *b*, such line will represent the development of one revolution of the helix; but since in orthographic projection one-half only of the screw is visible, the development required will be equal to the semi-circumference of the cylinder. Bisect *a b* in o; and draw o 4 parallel to *a c*. Then will o 4 represent the inclination or rise of the thread for half the circumference or visible portion of the screw.

ART. 167.—Upon the end of the axis or centre line of No. 2, describe a semicircle, equal in diameter to the diameter of the screw at the top of the thread; and divide the semicircle by radial lines into any number of equal parts, 1, 2, 3, 4. From the same centre, with a radius equal to that of the screw at the bottom of the thread, describe the semicircle 1′ 2′ 3′, representing the cylindrical part of the screw. Then will 1 1′ or 2 2′ represent the depth of the thread, which for a square-top-and-bottom-thread screw is generally equal to half the pitch. From 0, 1, 3, 4 draw lines parallel to the centre line or axis of the screw; and at any convenient distance draw *f* 0′ parallel thereto. Divide *f* 0′ into equal portions in 4′, 4″, &c., making each portion equal to half the pitch of the screw. If each division on the line *f* 0′ be now subdivided into the *same number* of parts as the semicircle in points 1, 2, 3, such points will correspond with the points, 1′, 2′, 3′, of development on the line 0 4, No. 1; because 0′ 4′, No. 2, is equal to 0 4, No. 1 (or half the pitch), which is divided into the same number of parts as the semicircle. Find, therefore, the projection of points 1, 2, 3, 4′ in the line *f* 0′, by Problem XLIX.; proceed in like manner with all the divisions 4′, 4″, &c., as shown by construction lines or projecting rays; and draw the curves. The projection of the short curve, or that portion of the thread which is in contact with the cylinder, is effected in like manner to that just described, —vertical lines being drawn from points 1′, 2′, 3′, &c. It will be seen that the return of the long curve from and above point *g* (as represented by the dotted line) is of the same character as below that point, and, if produced, would terminate at *h* in the line drawn from 4″: therefore the space included between 0′ and 4″ will be the measure of the pitch of the screw, or one revolution of the helix.

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* The pitch of a screw is the distance between the centre of one thread and that of the next succeeding thread, and is the measure of the screw's rectilinear motion for every revolution.

ART. 168.—No. 3 exhibits a vertical section of the nut, the mode of delineating which is the same as described for the screw: it is, therefore, only necessary to remark that the curved line or helix will incline in an opposite direction to the screw; the reason of which will be obvious on inspecting the dotted lines representing a continuation of the thread in the elevation No. 1. It may also be remarked that a portion of the long curve will be invisible, whereas the short curve will be entire.

Art. 169.—In finding the projection of a V-thread screw, the operation of projecting the curves is very similar to that described for the square thread. Having drawn a semicircle (see No. 4) equal in diameter to that of the screw at the top of the thread, and divided it into four equal parts, draw the right line *f′* 0′, and divide it into equal parts in points 8, 8′, 8″,—each division being equal to the pitch of the screw, as *c a*, No. 1, and subdivided in like manner into eight equal parts. Find the projection of the long curves for the top of the thread, which will commence at 0, and terminate at point 4′; and so on for each succeeding curve, commencing at points 8, 8′, &c. For the short curves, representing the bottom of the thread, commence by drawing a line from point 4, and so on, terminating at point 8; and then join the top and bottom of the thread by right lines, as shown in the Drawing, in which No. 5 represents a vertical section of the screw.

Art. 170.—In finding the projection of a screw with two threads, commonly called a double-thread screw (presuming the thickness of thread and space between to be the same as No. 2), it will only be necessary to take two divisions from the line *f* 0′, No. 2, instead of one, as for a single thread; in which case the first curve, instead of terminating at *g*, would terminate at *k* (see *k′*, No. 6); and for three threads the first curve would terminate at *i*; and so on for any number of threads,—observing to divide the space or rise of the thread, 4 *k* or 4 *i*, into the same number of equal parts as the plan or semi-circumference, 0, 1, 2, 3, 4.

Art. 171.—In preparing drawings of bolts for the workshop, as shown at Nos. 7 and 8, which represent a V and square thread screw, it is sufficient to project the threads into right lines in the following manner:—At right angles to the axis of the bolt draw *a b*, No. 7, equal to the diameter. From *a*, on the left-hand side of the bolt, set off *a c*, equal to half the pitch, and join *c b*, which will give the proper inclination of the thread for a single threaded screw. Upon the right line *a d*, set off the required number of divisions, equal to the pitch of the screw; and draw lines parallel to *c b*.

ART. 172.—The angle of the thread, as determined by Messrs. Whitworth and Co., in their improvements in screw threads, is 55°; to save time, however, the set square of 30° will be sufficient for the purpose when making working drawings. If, therefore, the set square of 30° be applied to the edge of the T square, and lines be drawn from the points of two consecutive threads, as *c* and *f*, the point of intersection at *g* will give the depth of the thread (the angle in this case being 60°);—the top and bottom of each thread being joined by right lines *c g*, *f g*. [See No. 7.]

It will be understood that we give the above as the simplest mode of finding approximately the depth of the thread; and in order that the student may be consistent as regards the number of threads for any given diameter of bolt, we have appended a table showing the number of threads, standard measure, for every inch in length of the screw.

DIAMETER IN INCHES.	THREADS TO THE INCH.	DIAMETER IN INCHES.	THREADS TO THE INCH.
3/16	24	2	4½
¼	20	2¼	4
5/16	18	2½	4
⅜	16	2¾	3½
7/16	14	3	3½
½	12	3¼	3¼
⅝	11	3½	3¼
¾	10	3¾	3
⅞	9	4	3
1	8	4¼	2⅞
1⅛	7	4½	2⅞
1¼	7	4¾	2¾
1⅜	6	5	2¾
1½	6	5¼	2⅝
1⅝	5	5½	2⅝
1¾	5	5¾	2½
1⅞	4½	6	2½

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